Northeastern University
Department of Mathematics

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MTH 3481—TOPOLOGY 3
Final Exam

This is a take-home exam, due Friday, June 6 (as hard copy in my mailbox), or Sunday, June 8 (as pdf file by email). Give complete proofs or justifications for each statement you make. Show all your work.

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1. Let M^n be a compact, connected, n-dimensional manifold (possibly non-orientable).
   (a) If n is odd, show that $\chi(M) = 0$.
   (b) If n is even, and M is the boundary of a compact, connected manifold W^{n+1}, show that $\chi(M)$ is even.

2. Let M^3 be a compact, connected 3-manifold.
   (a) Assume M is orientable. What are the possible homology groups of M? Be as specific as possible.
   (b) Assume M is non-orientable. Show that $H_1(M)$ is infinite. (Hint: Use problem 1(a).) What else can you say about the possible homology groups of M? Be as specific as possible.

3. Let M^n be a compact, connected, orientable, n-dimensional manifold. Let $f\colon M \to M$ be a map such that $f_*\colon H_n(M;\mathbb{Z}) \to H_n(M;\mathbb{Z})$ is an isomorphism. Show that the induced homomorphisms
$$f_*\colon H_i(M;G) \to H_i(M;G) \quad \text{and} \quad f^*\colon H^i(M;G) \to H^i(M;G)$$
are isomorphisms, for all $i \geq 0$, and all abelian groups G.

4. Let M^n be a compact, connected, orientable, n-dimensional manifold, and let $N^{n-1} \subset M^n$ a codimension-one compact, connected submanifold.
   (a) If $H_1(M;\mathbb{Z}) = 0$, then $M \setminus N$ has exactly two (path) components.
   (b) Is the conclusion true if $H_1(M;\mathbb{Z}) \neq 0$?