MTH3107 — TOPOLOGY 2 — WINTER 2003

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Homework 2

1. Problem 16 on page 132 of Hatcher's book.

2. Problem 17 on page 132 of Hatcher's book.

3. Show that the Hurewicz homomorphism is natural. That is, if $f\colon X \to Y$ is a (continuous) map, and $f(x_0) = y_0$, then the following diagram commutes:

$$\begin{array}{ccc} \pi_1(X, x_0) & \xrightarrow{f_\sharp} & \pi_1(Y, y_0) \\ \big\downarrow{\scriptstyle h_X} & & \big\downarrow{\scriptstyle h_Y} \\ H_1(X) & \xrightarrow{f_*} & H_1(Y) \end{array}$$

4. Let $p\colon E \to B$ be a covering map, $p(e_0) = b_0$. Recall that $p_\sharp\colon \pi_1(E, e_0) \to \pi_1(B, b_0)$ is a monomorphism (see e.g. Munkres' book, Theorem 54.6(a) on page 346). Question: Is $p_*\colon H_1(E) \to H_1(B)$ also a monomorphism? Give either a proof or a counterexample.

5. Let X be a space. Define the (unreduced) suspension of X to be
$$\Sigma X = X \times I/(X \times \{0\} \sim (x_0, 0) \text{ and } X \times \{1\} \sim (x_1, 1))$$
If $f\colon X \to Y$ is a (continuous) map, define $\Sigma f\colon \Sigma X \to \Sigma Y$ by $\Sigma f([x, t]) = [f(x), t]$.

   (a) Show that, for each $i \geq 0$, there is an isomorphism
   $$\widetilde{H}_i(X) \xrightarrow{\cong} \widetilde{H}_{i+1}(\Sigma X)$$

   (b) Show that the above "suspension" isomorphism is natural. That is, if $f\colon X \to Y$ is a map, then the following diagram commutes:
   $$\begin{array}{ccc} \widetilde{H}_i(X) & \xrightarrow{\cong} & \widetilde{H}_{i+1}(\Sigma X) \\ \big\downarrow{\scriptstyle f_*} & & \big\downarrow{\scriptstyle (\Sigma f)_*} \\ \widetilde{H}_i(Y) & \xrightarrow{\cong} & \widetilde{H}_{i+1}(\Sigma Y) \end{array}$$

   (See also problems 20 and 21, on page 132 of Hatcher's book.)