Northeastern University
Department of Mathematics

Prof. Alex Suciu **MTH 3107 – TOPOLOGY II** **Winter 1998**

Take-Home Final Exam

Due Tuesday, March 17

Instructions: *Do at least 6 of the following 10 problems. Give complete proofs or justifications for each statement you make. Show all your work.*

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1. Let $X = [0,1]$ and $A = \{1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots\} \cup \{0\}$. Is $H_1(X, A)$ isomorphic to $\widetilde{H}_1(X/A)$? Explain why, or why not.

2. (a) Let X and Y be two finite CW complexes. Show that $\chi(X \times Y) = \chi(X)\chi(Y)$.
   (b) Let A and B be two subcomplexes of X such that $X = A \cup B$. Show that $\chi(X) = \chi(A) + \chi(B) - \chi(A \cap B)$.

3. Let $f : (X, A) \to (Y, B)$ be a continuous map. Assume that $f : X \to Y$ and $f|_A : A \to B$ are homotopy equivalences.
   (a) Show that $f_* : H_*(X, A) \to H_*(Y, B)$ is an isomorphism.
   (b) Give an example where $H_*(X, A) \not\cong H_*(Y, B)$, although X is homotopy equivalent to Y, and A is homotopy equivalent to B.

4. Let $X = \mathbb{RP}^3 \times L(4, 1)$ be the product of the projective space $\mathbb{RP}^3 = S^3/\mathbb{Z}_2$ with the lens space $L(4, 1) = S^3/\mathbb{Z}_4$.
   (a) Find a CW-decomposition of X.
   (b) Determine the chain complex $(C_\bullet(X), d)$ associated to that cell decomposition.
   (c) Compute the homology groups $H_*(X)$.

5. Let $X = G_3(\mathbb{R}^6)$ be the Grassmanian of 3-planes in $\mathbb{R}^6$.
   (a) Find a CW-decomposition of X.
   (b) Determine the chain complex $(C_\bullet(X), d)$ associated to that cell decomposition.
   (c) Compute the homology groups $H_*(X)$.

6. Let T_g be the orientable surface of genus g.
   (a) Describe a 2-fold cover $p : T_3 \to T_2$.
   (b) Compute $p_* : H_1(T_3) \to H_1(T_2)$.
   (c) Compute $p_* : H_2(T_3) \to H_2(T_2)$.

7. Let $X = (\mathbb{C}^2 \setminus \{0\})/(z_1, z_2) \sim (2z_1, 2z_2)$. Compute $H_*(X)$.

8. Show that $\mathbb{RP}^2$ is not a retract of $\mathbb{RP}^3$.

9. Problem 1, in Bredon's book, p. 259.

10. Problem 5, in Bredon's book, p. 259.