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MTH 1230 **LINEAR ALGEBRA** **Spring 1999**

SAMPLE QUIZ 5

1. Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 4 & 5 & 2 \\ -1 & 3 & 8 \end{bmatrix}$.

(a) What is $\ker A$, and what is its orthogonal complement?

(b) What is $\ker A^{\top}$, and what is its orthogonal complement?

2. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$.

(a) What is $\ker A$, and what is its orthogonal complement?

(b) What is $\ker A^{\top}$, and what is its orthogonal complement?

3. Find the least squares solution of the inconsistent system $A\mathbf{x} = \mathbf{b}$ for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.

4. Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ -1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$.

(a) Find the least squares solution $\left[\begin{smallmatrix} \bar{x} \\ \bar{y} \end{smallmatrix}\right]$ of the inconsistent system $A \cdot \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] = \mathbf{b}$.

(b) Use your answer to part (a) to find the projection of $\mathbf{b}$ onto CS(A), the column space of A.

5. Find the equation of the least-squares line that fits the following data points:

x	1	2	4	5
y	0	1	2	3

Sketch the resulting line. What is the predicted value of y at $x = 6$, based on this model?

6. A company gathers the following data:

Year	1991	1992	1993	1994	1995	1996	1997
Annual Sales (in millions of dollars)	1.2	2.8	3.6	4.5	6	7.5	8.2

Represent the years $1991, \dots, 1997$ as $-3, -2, -1, 0, 1, 2, 3$, respectively, and let x denote the year. Let y denote the annual sales (in millions of dollars).

(a) Find the least squares line relating x and y.

(b) Use the equation obtained in part (a) to estimate the annual sales for the year 2000.

7. Let $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.

(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A.

(c) Find a basis for each eigenspace of A.

(d) Find a diagonal matrix Λ and an invertible matrix S such that $A = S \cdot \Lambda \cdot S^{-1}$.

8. Let A be a 3×3 matrix, with eigenvalues $\lambda_1 = -2$, $\lambda_2 = 0$, $\lambda_3 = 5$.

(a) Compute $\operatorname{tr}(A)$ and $\det(A)$.

(b) Is A invertible? Explain your answer.

(c) Is A diagonalizable? Explain your answer.

(d) Compute $\operatorname{tr}(A^3)$ and $\det(A^3)$.

9. Let $A = \begin{bmatrix} 4 & -7 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) Find the eigenvalues of A.

(b) Find a basis for each eigenspace of A.

(c) Find a diagonal matrix Λ and an invertible matrix S such that $A = S \cdot \Lambda \cdot S^{-1}$.

10. A 2×2 matrix A has first row $\begin{bmatrix} -2 & 5 \end{bmatrix}$ and eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$.

(a) Find A.

(b) What are the eigenvalues of A^{-1}?

(c) Compute $\det(A^{-1} + I)$, where I is the identity 2×2 matrix. Explain your result.

11. A 4×4 matrix has eigenvalues $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3, \lambda_4 = 4$.

(a) Find the eigenvalues of A^2.

(b) Find the trace of A^2.

(c) Find the determinant of A^2.

12. Let $A = \begin{bmatrix} -6 & 11 \\ 1 & 4 \end{bmatrix}$.

(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A.

(c) Find a basis for each eigenspace of A.

(d) Find a diagonal matrix Λ and an invertible matrix S such that $A = S \cdot \Lambda \cdot S^{-1}$.

13. Let A be a 4×4 matrix, with eigenvalues $\lambda_1 = -1$, $\lambda_2 = 2$, $\lambda_3 = 2$, $\lambda_4 = 3$.

(a) Compute $\operatorname{tr}(A)$ and $\det(A)$.

(b) Is A invertible? Always? Sometimes? Never? Explain your answer.

(c) Is A diagonalizable? Always? Sometimes? Never? Explain your answer.

(d) Compute $\operatorname{tr}(A^2)$ and $\det(A^2)$.