Prof. A. Suciu

MTH 1230 **LINEAR ALGEBRA** **Spring 1999**

SAMPLE QUIZ 3

1. Let $A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 2 & 2 & 0 \\ -6 & -3 & 0 & 1 \\ 2 & 1 & -2 & 0 \end{bmatrix}$.

(a) Find a basis for $\operatorname{im} A$.

(b) Find a basis for $\ker A$.

(c) Find $\operatorname{rank} A$.

2. Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 4 & 5 & 2 \\ -1 & 3 & 8 \end{bmatrix}$.

(a) Determine whether the column vectors of A are dependent or independent. If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

(b) Find $\ker A$ and $\operatorname{im} A$.

(c) Does the equation $A \cdot \vec{x} = \vec{b}$ have a solution for every choice of $\vec{b}$ in $\mathbb{R}^3$? Explain your answer.

3. Are the following vectors independent or dependent? If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 6 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} -5 \\ 7 \\ -3 \end{bmatrix}$$

4. In each of the following, a subset S of $V = \mathbb{R}^3$ is given. Circle one answer:

(a) $S = \{(t, 2t, 3t) \mid t \text{ is a real number}\}$

S is closed under addition:	YES	NO	MAYBE
S is closed under scalar multiplication:	YES	NO	MAYBE
S is a vector subspace of V:	YES	NO	MAYBE

(b) $S = \{(t, 2t, 3t) \mid t \text{ is a positive real number}\}$

S is closed under addition:	YES	NO	MAYBE
S is closed under scalar multiplication:	YES	NO	MAYBE
S is a vector subspace of V:	YES	NO	MAYBE

(c) $S = \{(t, 2t, 3t) \mid t \text{ is an integer}\}$

S is closed under addition:	YES	NO	MAYBE
S is closed under scalar multiplication:	YES	NO	MAYBE
S is a vector subspace of V:	YES	NO	MAYBE

(d) $S = \{(t+1, 2t, 3t-1) \mid t \text{ is a real number}\}$

S is closed under addition:	YES	NO	MAYBE
S is closed under scalar multiplication:	YES	NO	MAYBE
S is a vector subspace of V:	YES	NO	MAYBE