Prof. A. Suciu

MTH 1230 **LINEAR ALGEBRA** **Spring 1999**

QUIZ 1

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1. $\boxed{\text{10 points}}$

(a) Use Gauss-Jordan elimination (rref) to solve the following system of equations. Carry out the elimination all the way, before solving.

$$\begin{aligned} x_1 + x_2 + x_3 + 2x_4 \qquad &= 1 \\ 2x_1 + 2x_2 + 3x_3 + x_4 + 4x_5 &= 3 \\ -x_1 - x_2 \qquad - 5x_4 + 4x_5 &= 0 \end{aligned}$$

(b) Explain why there are no solutions, an infinite number of solutions (write down the general solution in vector form if this is the case), or exactly one solution (write down the solution in this case).

2. $\boxed{\text{6 points}}$ The reduced row echelon forms of the augmented matrices of 3 systems are given below. In each case, indicate the rank of the matrix of coefficients (to the left of the dotted lines), and the number of solutions of the system (you need **not** write down the solutions.)

(a) $\begin{bmatrix} 1 & 2 & 3 & \vdots & 4 \\ 0 & 0 & 0 & \vdots & 1 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 2 & \vdots & 4 \\ 0 & 0 & 1 & \vdots & 2 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 0 & \vdots & 4 \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$

3. $\boxed{\text{2 points}}$ Find the matrix A of the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by

$$\begin{aligned} y_1 &= 2x_1 + 3x_2 - x_3 + x_4 \\ y_2 &= -x_3 + 4x_4 \\ y_3 &= 5x_1 - x_2 + 2x_3 \end{aligned}$$

4. $\boxed{\text{4 points}}$ Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$, where

$$T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}6\\-1\end{bmatrix}, \qquad T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}7\\12\end{bmatrix}, \qquad T\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-2\\5\end{bmatrix}.$$

(a) Find the matrix A of T.

(b) Compute $A \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} =$

5. $\boxed{\text{4 points}}$ Sketch the image of the unit square under the linear transformation

$$T(\vec{x}) = \begin{bmatrix}1 & -1\\3 & 2\end{bmatrix}\vec{x}.$$

6. $\boxed{\text{4 points}}$ Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, which is a counter-clockwise rotation of 30°, followed by a dilation by a factor of 2. Give the matrix A of T.