Prof. A. Suciu

MTH 1230 **LINEAR ALGEBRA** **Spring 2001**

SAMPLE EXAM 4

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1. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$. What is the area of the parallelogram spanned by the column vectors of $5I_2 - A$?

2. Compute the area of the triangle with vertices at $(1, 2)$, $(5, -7)$, $(-3, 8)$.

3. Compute the area of the quadrilateral with vertices at $(1, 2)$, $(5, -7)$, $(-3, 8)$, $(10, 10)$.

4. Find the volume of the 3-paralleliped defined by the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \qquad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \qquad \vec{v}_3 = \begin{bmatrix} 5 \\ 1 \\ 3 \\ -4 \end{bmatrix}.$$

5. Find a 2×2 matrix A such that $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ are eigenvectors of A, with eigenvalues -2 and 4, respectively.

6. A 5×5 matrix A has eigenvalues $\lambda_1 = -1$, $\lambda_2 = 2$, $\lambda_3 = 2$, $\lambda_4 = 3$, $\lambda_5 = 4$.

(a) Compute: $\operatorname{tr} A =$

(b) Compute: $\det A =$

(c) Compute: $\det(3I_5 - A) =$

(d) Is A invertible? Why, or why not?

(e) Is A orthogonal? Why, or why not?

7. Let A and B be two 3×3 matrices, with $\det A = -2$ and $\det B = 0$.

(a) Is A invertible? If yes, compute $\det(A^{-1})$. If not, say so.

(b) Is B invertible? If yes, compute $\det(B^{-1})$. If not, say so.

(c) Compute: $\det(4A) =$

(d) Compute: $\det(A^4) =$

8. Let A be a 3×3 matrix, with eigenvalues $\lambda_1 = -2$, $\lambda_2 = 0$, $\lambda_3 = 5$.

(a) Compute $\operatorname{tr}(A)$ and $\det(A)$.

(b) Is A invertible? Explain your answer.

(c) Is A diagonalizable? Explain your answer.

(d) Compute $\operatorname{tr}(A^3)$ and $\det(A^3)$.

9. Let $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 9 & -5 \end{bmatrix}$.

(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A.

(c) Find a basis for each eigenspace of A.

10. Let $A = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$.

(a) Find the characteristic equation for A.

(b) Find the eigenvalues of A.

(c) Find a basis for each eigenspace of A.

(d) Form a matrix S using the two independent eigenvectors from part () as column vectors, and calculate S^{-1}.

(e) Calculate $S^{-1} \cdot A \cdot S$. Explain your answer.

11. Let $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.

(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A.

(c) Find a basis for each eigenspace of A.

(d) Find a diagonal matrix Λ and an invertible matrix S such that $A = S \cdot \Lambda \cdot S^{-1}$.
[You do not have to calculate S^{-1}.]