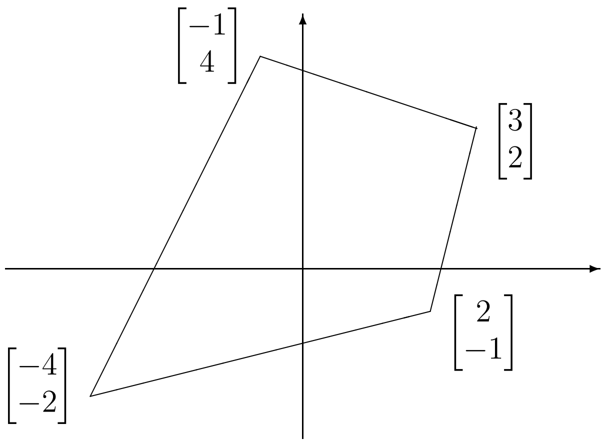
Prof. Alexandru Suciu

MTH 1230 **LINEAR ALGEBRA** **Spring 2001**

EXAM 4

1. $\boxed{\text{12 pts}}$

(a) Compute the area of the region enclosed by the folowing quadrilateral:



(b) Compute the area of the parallelogram spanned by the vectors $\begin{bmatrix}2\\2\\1\\3\end{bmatrix}$ and $\begin{bmatrix}0\\1\\1\\1\end{bmatrix}$.

2. $\boxed{\text{7 points}}$ Let A and B be two 5×5 matrices, with $\det A = 0$ and $\det B = -3$.

(a) Is A invertible? Why, or why not?

(b) Is A orthogonal? Why, or why not?

(c) Is B invertible? Why, or why not?

(d) Is B orthogonal? Why, or why not?

(e) Compute: $\det(B \cdot A \cdot B) =$

(f) Compute: $\det(B^\top)^3 =$

(g) Compute: $\det(2B) =$

3. $\boxed{\text{8 points}}$ Find a 2×2 matrix A such that $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -5 \end{bmatrix}$ are eigenvectors of A, with eigenvalues -7 and 3, respectively.

4. $\boxed{\text{12 points}}$ A 4×4 matrix A has eigenvalues $\lambda_1 = -3$, $\lambda_2 = -2$, $\lambda_3 = 1$, $\lambda_4 = 4$.

(a) What is the characteristic polynomial of A?

(b) Compute $\operatorname{tr}(A)$ and $\det(A)$.

(c) What are the eigenvalues of A^2?

(d) Compute $\operatorname{tr}(A^2)$ and $\det(A^2)$.

(e) Compute $\det(A + 2I_4)$

(f) Is A invertible? If yes, compute $\det(A^{-1})$. If not, explain why not.

(g) Is A diagonalizable? If yes, compute its diagonalization D. If not, explain why not.

5. $\boxed{\text{12 points}}$ Let $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & -3 \\ 0 & 1 & 8 \end{bmatrix}$.

(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A.

(c) Find a basis for each eigenspace of A.

(d) Find an invertible matrix S and a diagonal matrix D such that $A = S \cdot D \cdot S^{-1}$.
[You do not have to calculate S^{-1}.]