Prof. Alexandru Suciu

MTH 1230 **LINEAR ALGEBRA** **Spring 2001**

EXAM 2

1. 12 points

Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ -1 & 0 & 1 & 2 & 3 \\ 2 & 3 & 0 & 5 & 8 \end{bmatrix}$.

(a) Find a basis for the image of A.

(b) Find a basis for the kernel of A.

(c) Find the rank and the nullity of A.

2. [16 points] Consider the folowing four vectors in $\mathbb{R}^4$.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \qquad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \qquad \vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 3 \end{bmatrix}, \qquad \vec{v}_4 = \begin{bmatrix} 2 \\ 1 \\ 7 \\ 4 \end{bmatrix}.$$

Also let A be the 4×4 matrix with columns $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$.

(a) Are the vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ independent or dependent? If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

(b) Do the vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ form a basis for $\mathbb{R}^4$? Explain your answer.

(c) Does the equation $A \cdot \vec{x} = \vec{0}$ only have the solution $\vec{x} = \vec{0}$, or does it have other solutions? Explain your answer.

(d) Does the equation $A \cdot \vec{x} = \vec{b}$ have a solution for every choice of $\vec{b}$ in $\mathbb{R}^4$? Explain your answer.

3. [10 points] Let V be the subspace of $\mathbb{R}^3$ defined by the equation $x_1 + 2x_2 - 5x_3 = 0$.

(a) Find a basis for V.

(b) Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $\ker T = \{\vec{0}\}$ and $\operatorname{im} T = V$. Describe T by its matrix A.

4. [12 points] In each of the following, a subset V of $\mathbb{R}^3$ is given. Circle one answer:

(a) $V = \left\{ \begin{bmatrix} x+y+z \\ x+z \\ y \end{bmatrix} \;\middle|\; x, y, z \text{ arbitrary constants} \right\}$

Is closed under addition:	YES	NO
Is closed under scalar multiplication:	YES	NO
Is a vector subspace of $\mathbb{R}^3$:	YES	NO

(b) $V = \left\{ \begin{bmatrix} x+y+z \\ x+z \\ y+1 \end{bmatrix} \;\middle|\; x, y, z \text{ arbitrary constants} \right\}$

Is closed under addition:	YES	NO
Is closed under scalar multiplication:	YES	NO
Is a vector subspace of $\mathbb{R}^3$:	YES	NO

(c) $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \;\middle|\; x, y, z \text{ positive integers} \right\}$

Is closed under addition:	YES	NO
Is closed under scalar multiplication:	YES	NO
Is a vector subspace of $\mathbb{R}^3$:	YES	NO

(d) $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \;\middle|\; xy \leq 0 \right\}$

Is closed under addition:	YES	NO
Is closed under scalar multiplication:	YES	NO
Is a vector subspace of $\mathbb{R}^3$:	YES	NO