

**EXAM 1**

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1. 10 points

- (a) Use Gauss-Jordan elimination (rref) to solve the following system of equations. Carry out the elimination all the way, before solving.

$$\begin{aligned}x_1 + 2x_2 + 3x_3 + 4x_4 &= 5 \\2x_1 + 5x_2 + 7x_3 + 11x_4 &= 12 \\x_2 + x_3 + 4x_4 &= 3\end{aligned}$$

- (b) Explain why there are no solutions, an infinite number of solutions (write down the general solution in vector form if this is the case), or exactly one solution (write down the solution in this case).

2. 9 points Let  $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$  and  $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ .

(a) Use Gauss-Jordan elimination to compute the inverse of the matrix  $A$ .

(b) Use the answer in part (a) to solve the system  $A\vec{x} = \vec{b}$ . (No credit for any other method!)

3. 6 points The reduced row echelon forms of the augmented matrices of 3 systems are given below. In each case, indicate the rank of the matrix of coefficients (to the left of the dotted lines), and the number of solutions of the system (you need **not** write down the solutions.)

(a) 
$$\begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 0 & 0 & \vdots & 2 \\ 0 & 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 0 & 1 & \vdots & 0 \end{bmatrix}$$

(b) 
$$\begin{bmatrix} 1 & 2 & 0 & 0 & \vdots & 2 \\ 0 & 0 & 0 & 0 & \vdots & 1 \\ 0 & 0 & 1 & 3 & \vdots & 0 \\ 0 & 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

(c) 
$$\begin{bmatrix} 1 & 2 & 3 & 0 & \vdots & 2 \\ 0 & 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & 1 & \vdots & 3 \\ 0 & 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

4. 3 points Find the matrix  $A$  of the linear transformation  $T : \mathbb{R}^5 \rightarrow \mathbb{R}^3$  given by

$$y_1 = x_2 - 2x_3 + x_4$$

$$y_2 = -x_1 + x_3 + x_4 - x_5$$

$$y_3 = 2x_1 - x_2 + 3x_5$$

5. 3 points Write down (in format as above) the linear transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by

$$T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ 6 \end{bmatrix}, \quad T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$

6. 6 points Find the matrix  $A$  of the linear transformation  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  with

$$T \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad T \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}.$$

7. 4 points Sketch the image of the unit square under the linear transformation

$$T(\vec{x}) = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix} \vec{x}.$$

8. 9 points Find the matrices of the following linear transformations:

(a)  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ , a counter-clockwise rotation of  $60^\circ$ , followed by a dilation by a factor of 3.

(b)  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ , the reflection in the  $y$ - $z$ -plane.

(c)  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ , the projection onto the  $y$ - $z$ -plane.