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MTH 1187–Probability **SOLUTIONS to EXAM 3** **Winter 2001**

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(1) A random variable X has $E(X) = -4$ and $E(X^2) = 30$. Let $Y = -3X + 7$. Compute:

(a) $V(X) = E(X^2) - E(X)^2 = \boxed{14}$

(b) $V(Y) = (-3)^2 V(X) = \boxed{126}$

(c) $E((X+5)^2) = E(X^2 + 10X + 25) = E(X^2) + 10E(X) + 25 = \boxed{15}$

(d) $E(Y^2) = V(Y) + E(Y)^2 = 126 + (-3E(X) + 7)^2 = \boxed{487}$

(2) A deck has only face cards: 4 Kings, 4 Queens, and 4 Jacks. Two cards are drawn at random, without replacement. If Q is the number of Queens obtained, find the expected value, the variance, and the standard deviation of Q.

$$P(Q=0) = \frac{8}{12}\cdot\frac{7}{11} = \frac{14}{33}$$

$$P(Q=2) = \frac{4}{12}\cdot\frac{3}{11} = \frac{1}{11}$$

$$P(Q=1) = 1 - \frac{14}{33} - \frac{1}{11} = \frac{16}{33}$$

$$E(Q) = 1\cdot\frac{16}{33} + 2\cdot\frac{1}{11} = \boxed{\frac{2}{3}}$$

$$E(Q^2) = 1^2\cdot\frac{16}{33} + 2^2\cdot\frac{1}{11} = \frac{28}{33}$$

$$V(Q) = \frac{28}{33} - \left(\frac{2}{3}\right)^2 = \boxed{\frac{40}{99}}$$

$$D(Q) = \frac{2}{3}\sqrt{\frac{10}{11}} \simeq \boxed{0.635642}$$

(3) In a certain casino game, you can win either \$5, with probability 0.05, or \$2, with probability 0.2, or lose \$1, with probability 0.75.

(a) Find the mean and variance of your net winnings if you play once.

$$E(X) = 5\cdot 0.05 + 2\cdot 0.2 - 1\cdot 0.75 = \boxed{-0.1}$$

$$E(X^2) = 5^2\cdot 0.05 + 2^2\cdot 0.2 + (-1)^2\cdot 0.75 = 2.8$$

$$V(X) = 2.8 - (-0.1)^2 = \boxed{2.79}$$

(b) Suppose you play 80 times this game. Find the mean and standard deviation of your total net winnings.

$$E(S_{80}) = 80\cdot E(X) = \boxed{-8}$$

$$V(S_{80}) = 80\cdot V(X) = 223.2, \qquad D(S_{80}) = \boxed{14.9399}$$

(c) Use Gaussian approximation to the probability you come out <u>ahead</u> after playing 80 times.

$$P(S_{80} > 0) = P\left(Z > \frac{0-(-8)}{14.9399}\right) = P(Z > 0.53548) = 0.5 - 0.20384 = \boxed{0.29616}$$

(4) A biased coin comes up heads 30% of the time. The coin is tossed 400 times. Let X be the number of heads in the 400 tossings.

(a) Use Chebyshev's inequality to bound the probability that X is between 100 and 140.

$$n = 400, \qquad p = 0.3, \qquad \mu = np = 120, \qquad \sigma^2 = np(1-p) = 84$$

$$P(100 \le X \le 140) = P(|X - 120| \le 20) \ge 1 - \frac{84}{20^2} = \boxed{0.79}$$

(b) Use Gaussian approximation to compute the probability that X is between 100 and 140.

$$P(100 \le X \le 140) \simeq P\left(\frac{100-120}{\sqrt{84}} \le Z \le \frac{100-120}{\sqrt{84}}\right) = P(-2.18218 \le Z \le 2.18218) \simeq \boxed{0.9709}$$