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MTH 1125 | **Calculus 3** | **Spring 2002**

SOLUTIONS TO QUIZ 7

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1. Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^{3/4}}$. For what values of x does the series converge absolutely, converge conditionally, or diverge?

(a) Find the center of the series.

$$a = 3$$

(b) Find the radius of convergence of the series.

$$R = \lim_{n\to\infty} \frac{\frac{1}{n^{3/4}}}{\frac{1}{(n+1)^{3/4}}} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{3/4} = 1^{3/4} = 1$$

(c) Test for convergence at the end-points of the interval of convergence.

Interval of (absolute) convergence: $a - R < x < a + R \implies 2 < x < 4$

at $x = 4$: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/4}}$ diverges (p-series with $p = \frac{3}{4} < 1$)

at $x = 2$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/4}}$ converges conditionally (alternating series with $\frac{1}{n^{3/4}} \searrow 0$)

(d) Finally, organize your answer, as follows:

- Series converges absolutely for: $\underline{2 < x < 4}$
- Series converges conditionally for: $\underline{x = 2}$
- Series diverges elsewhere.

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2. From the definition, find the degree 3 Taylor polynomial for $f(x) = \ln x$, centered at $a = 1$.

We have: $f'(x) = x^{-1}$, $f''(x) = -x^{-2}$, $f'''(x) = 2x^{-3}$. Hence:

$$\begin{aligned} P_3(x) &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 \\ &= 0 + 1\cdot(x-1) + \tfrac{(-1)}{2!}(x-1)^2 + \tfrac{2}{3!}(x-1)^3 \\ &= (x-1) - \tfrac{1}{2}(x-1)^2 + \tfrac{1}{3}(x-1)^3 \end{aligned}$$

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3. Recall that $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots + (-1)^n \dfrac{x^{2n}}{(2n)!} + \cdots$, for all real x. Find:

(a) The degree 4 Maclaurin polynomial for $\cos(\sqrt{2x})$.

$$P_4(x) = 1 - \frac{\left(\sqrt{2x}\right)^2}{2!} + \frac{\left(\sqrt{2x}\right)^4}{4!} - \frac{\left(\sqrt{2x}\right)^6}{6!} + \frac{\left(\sqrt{2x}\right)^8}{8!} = 1 - x + \tfrac{1}{6}x^2 - \tfrac{1}{90}x^3 + \tfrac{1}{2520}x^4$$

(b) The degree 4 Maclaurin polynomial for $\dfrac{\cos x - 1}{x^2}$.

$$P_4(x) = \frac{\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right) - 1}{x^2} = -\tfrac{1}{2} + \tfrac{1}{24}x^2 - \tfrac{1}{720}x^4$$