Prof. Alexandru Suciu

MTH 1125 **Calculus 3** **Spring 2002**

SOLUTIONS to QUIZ 4

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1. 5 points Write down the first 4 terms of the following series. Does the series converge, or does it diverge? Find its sum if it converges. **Explain!**

$$\sum_{n=1}^{\infty} \frac{2}{3^n}$$

The series starts as:

$$\sum_{n=1}^{\infty} \frac{2}{3^n} = \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \cdots$$

This is a geometric series, with initial term $a = \frac{2}{3}$ and ratio $r = \frac{1}{3}$. Since $|r| = \frac{1}{3} < 1$, the series **converges**, and its sum is

$$\frac{a}{1-r} = \frac{\frac{2}{3}}{1 - \frac{1}{3}} = 1$$

2. 5 points Write down the first 4 terms of the following series. Does the series converge, or does it diverge? Find its sum if it converges. **Explain!**

$$\sum_{n=2}^{\infty} \left(\frac{1}{\ln(n)} - \frac{1}{\ln(n+1)} \right)$$

The series starts as:

$$\left(\frac{1}{\ln 2} - \frac{1}{\ln 3}\right) + \left(\frac{1}{\ln 3} - \frac{1}{\ln 4}\right) + \left(\frac{1}{\ln 4} - \frac{1}{\ln 5}\right) + \left(\frac{1}{\ln 5} - \frac{1}{\ln 6}\right) + \cdots$$

This is a telescoping series. The n-th partial sum is

$$s_n = \frac{1}{\ln 2} - \frac{1}{\ln(n+1)}$$

and this sequence converges, since $\lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$. Hence the series **converges**, and its sum is:

$$\frac{1}{\ln 2} \simeq 1.4427$$

3. 5 points Does the following series converge, or does it diverge? **Explain!**

$$\sum_{n=1}^{\infty} \frac{2n}{10n+1}$$

We have:

$$\lim_{n\to\infty} \frac{2n}{10n+1} = \lim_{n\to\infty} \frac{2}{10+\frac{1}{n}} = \frac{1}{5}$$

Since this limit **does not equal** 0, the series itself **diverges**, by the Divergence Test (a.k.a. the n-th Term Test).

4. 5 points For which values of x does the following series converge? Find the sum of the series when it converges.

$$\sum_{n=0}^{\infty} \left(\frac{x-3}{2}\right)^n$$

This is a geometric series, with initial term $a = 1$ and ratio $r = \dfrac{x-3}{2}$. The series converges precisely when $|r| < 1$, that is:

$$\left|\frac{x-3}{2}\right| < 1 \iff -2 < x-3 < 2 \iff 1 < x < 5$$

For x in the interval $(1,5)$, the sum of the series is:

$$\frac{a}{1-r} = \frac{1}{1-\dfrac{x-3}{2}} = \frac{2}{5-x}$$