Prof. A. Suciu

MTH 1108 **Fundamentals of Integral Calculus** **Spring 1999**

QUIZ 5

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1. Solve the differential equation

$$yy' = e^{8x}$$

by separating the variables. Then determine the solution $y = y(x)$ for which $y(0) = 3$. **(9)**

2. Find all the values of k for which the function $y(x) = e^{kx}$ is a solution to the differential equation $y'' - 5y' + 6y = 0$. **(7)**

3. A glass of lemonade at 35°F is taken out of a refrigerator and brought into a room that has constant temperature 70°F. After 2 minutes, the temperature of the lemonade rises to 45°F. Suppose Newton's law of cooling applies. **(14)**

(a) What differential equation describes the rate of warming of the lemonade?

(b) Find the temperature $y(t)$ of the lemonade at time t minutes after is was brought into the room.

(c) What is the temperature of the lemonade, 5 minutes after is was brought into the room?

(d) What is the rate of warming of the lemonade, 5 minutes after is was brought into the room?

Table of Derivatives

$$(fg)' = f'g + fg' \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \qquad f(g(x))' = f'(g(x)) \cdot g'(x)$$

$$(x^n)' = nx^{n-1} \qquad (e^x)' = e^x \qquad (\ln x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x \qquad (\cos x)' = -\sin x \qquad (\tan x)' = \sec^2 x$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \qquad (\arctan x)' = \frac{1}{x^2+1}$$

Table of Antiderivatives

$$\int a\,dx = ax + C \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad \int e^{ax}\,dx = \frac{1}{a}\,e^{ax} + C \qquad (a \neq 0)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\,\cos(ax) + C \qquad \int \cos(ax)\,dx = \frac{1}{a}\,\sin(ax) + C \qquad (a \neq 0)$$

Differential Equations

Solution of $y' = ky$: $\quad y = Ce^{kt} \quad$ or $\quad y = 0$

(Exponential growth if $k > 0$, exponential decay if $k < 0$)

Solution of $y' = k(r - y)$: $\quad y = r + Ce^{-kt} \quad$ or $\quad y = r$

(Newton's law of cooling)

Solution of $y' = ky(r - y)$: $\quad y = \dfrac{r}{1 + Ce^{-rkt}} \quad$ or $\quad y = 0,\ y = r$

(Logistic equation)