MIDTERM

Instructions: Put your name in the blanks above. Put your final answers to each question in the designated spaces on these pages. Show your work—if there is not enough room, use another sheet.

---

(1) SET UP a linear program to solve the following problem. Be sure to identify the action variables, **ALL** the constraints, and the objective function. **DO NOT SOLVE**.

A health food store is making a tasty blend of carrots and avocados for a special diet. Carrots contain 100 units of vitamin A and 25 units of vitamin B per pound. Avocados contain 30 units of vitamin A and 80 units of vitamin B per pound. The mixture is to have at least 1,000 units, but no more than 3,000 units of vitamin A; and at least 1,200 units, but no more than 2,400 units of vitamin B. Carrots cost 60 cents per pound, and avocados cost \$1.10 per pound. How many pounds of each should be used in order to minimize costs?

(2) Use the substitution method to solve the following system of equations:

$$8x + 2y = 3$$
$$-4x + 4y = 11$$

(3) Solve the following system of linear inequalities **AND** shade the region:

$$y \geq 1, \qquad x \geq -2, \qquad 2x + 5y \leq 10$$
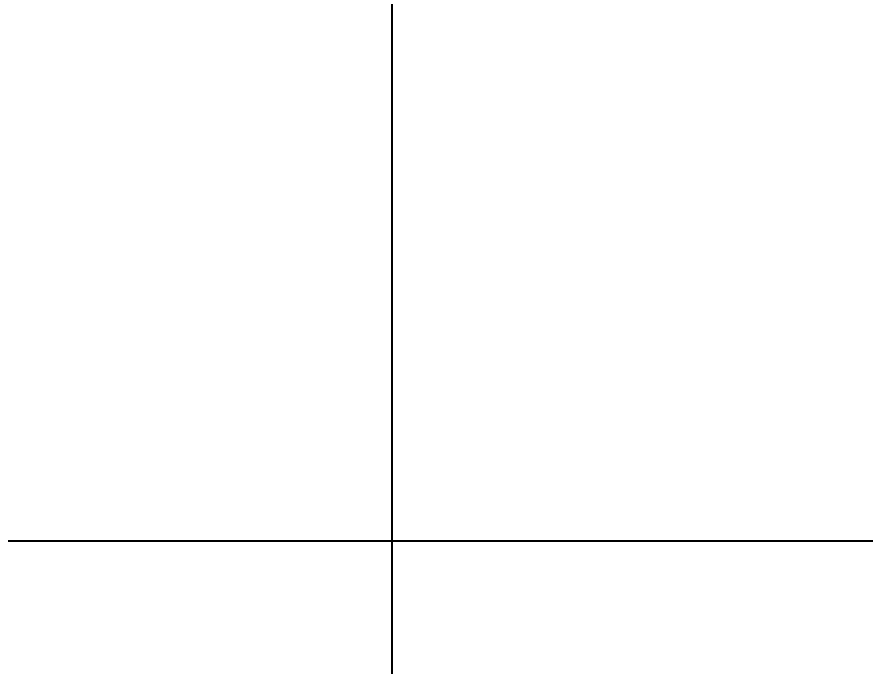
(4) Find $4A - B$, where $A = \begin{bmatrix} -2 & 0 \\ 1 & -5 \\ 7 & -1 \\ 4 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 9 & -6 \\ -7 & 4 \end{bmatrix}$.
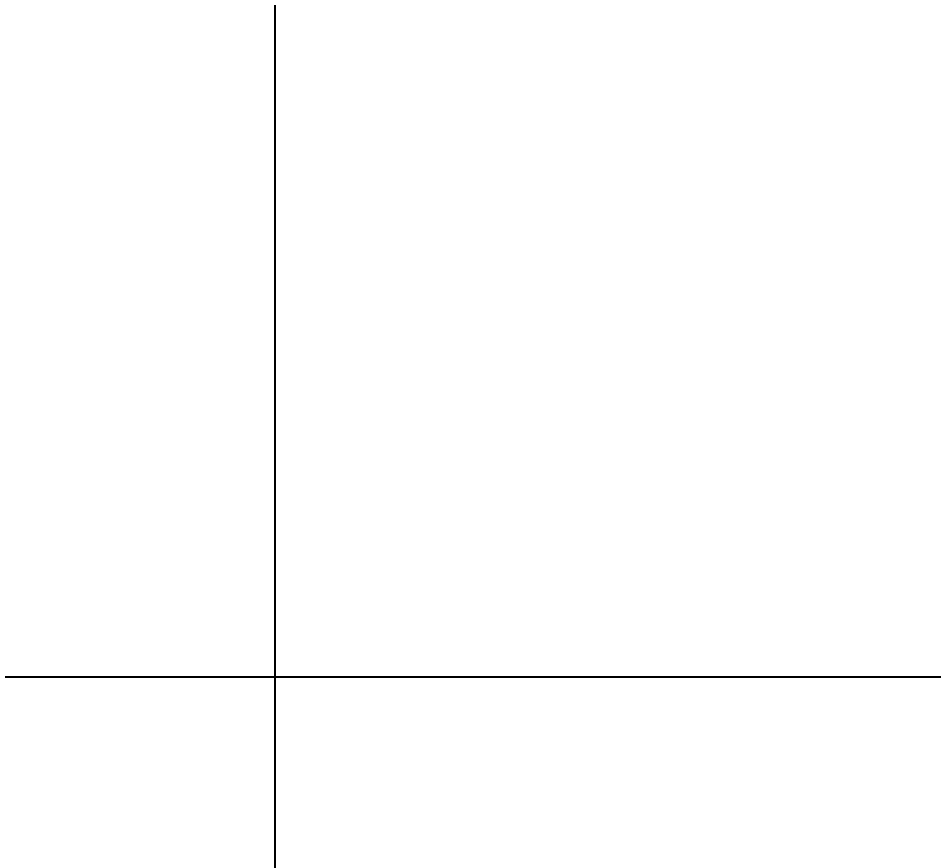
(5) Find the product

$$\begin{bmatrix} 1 & 4 \\ 3 & -2 \\ 5 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 0 \\ -3 & 4 & -5 \end{bmatrix} =$$

(6) Find the inverse of the matrix $A = \begin{bmatrix} 9 & 8 \\ -5 & -4 \end{bmatrix}$

(7) SOLVE the following linear programming problem. Be sure to shade in the feasible set, and mark down its corners (together with their coordinates).

Find the maximum and minimum values of $F = -3x + 5y$, subject to the constraints

$$y \geq 0, \quad y \leq 2, \quad -2x + y \leq 0, \quad 2x + y \leq 6.$$

Answer:

- The maximum value of F equals _____, and it occurs at the point (_____ , _____).

- The minimum value of F equals _____, and it occurs at the point (_____ , _____).

(8) The message

$$[-16, 12, -10, 10, -64, 51, -109, 89, -122, 99]$$

was encoded using the matrix $M = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix}$ and the coding scheme

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	−1	2	−2	3	−3	4	−4	5	−5	6	−6	7	−7	8	−8
Q	R	S	T	U	V	W	X	Y	Z	blank	'	,	.	!	?
9	−9	10	−10	11	−11	12	−12	13	−13	14	−14	15	−15	16	−16

(a) What matrix is needed for decoding the message?

(b) What is the message?

The message is: _____