

① A OS.

For Affine

$$0 \rightarrow A_0 \xrightarrow{a_0} A_1 \xrightarrow{a_1} \dots \xrightarrow{a_n} A_n \rightarrow 0$$

$$I = \left\langle dx_s, e_T \mid \begin{matrix} s = \text{circuits} \\ nH = \emptyset \\ H \in T \end{matrix} \right\rangle$$

Take  $a \in A_1 = E_1$ ;  $a^2 = 0$

$$H^*(A, a) = ?$$

$$A = \langle e_1, \dots, e_n \rangle = \left\langle \left[ \frac{dx_i}{\alpha_i} \right], \dots, \left[ \frac{dx_n}{\alpha_n} \right] \right\rangle$$

② Motivations 1)  $a \in E_1$ ,  $a$  is regular <sup>on A</sup> if  $H^*(A, a) = 0$

otherwise  $a$  is singular

2) Relations to local Coefficients.

$$0 \rightarrow \underbrace{\Sigma^0}_U \xrightarrow{d+w_1} \underbrace{\Sigma^1}_U \xrightarrow{d+w_2} \dots \xrightarrow{d+w_r} \underbrace{\Sigma^r}_U \rightarrow 0$$

$w \in \Sigma^1 \rightarrow A^0 \rightarrow A^1 \rightarrow A^2 \rightarrow \dots \rightarrow A^r$

$\sum c_{i_1, \dots, i_r} dx_{i_1} \wedge \dots \wedge dx_{i_r}$

skew comm. rule

under the embedding  $a = w_a$

$$= \sum a_i e_i = \sum a_i \frac{dx_i}{\alpha_i}$$

$$(d + w_a \wedge) \frac{dx}{\alpha} = w_a \wedge \frac{dx_i}{\alpha_i} = a \cdot e_i$$

$$(d + w_a \wedge) w_b = a \cdot b$$

$b \in A$

Define ;  $\mathcal{R} = \text{Boolean} \Rightarrow i$  is quasi-iso.  
 if  $a_i \notin \mathbb{N} \Rightarrow i^*$  is iso.  
 ↗ doesn't make sense

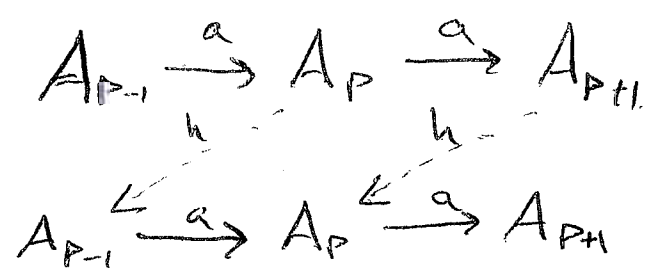
Thm [ESV] + [STV]

(Proj. Ann.)  $i$  is quasi-iso if

$$\sum_{H_i \rightarrow X} a_i \notin \mathbb{N} \quad \forall x \in L \Rightarrow \mathcal{R}_x \text{ is irred.}$$

③ "Generic"  $a \in A$ ,  $\rightsquigarrow$  (vanishing)

a) Central,  $\sum_{i=1}^n a_i \neq 0 \Rightarrow H^*(A, a) = 0$



If  $ha + ah = 1$  then  $H^*(A, a) = 0$ .

Here  $h = d = \sum (-1)^{j-1} e_{S_j}$

Pf 1 Well-defined c) Prove  $d \cdot a + a \cdot d = \sum_{i=1}^n a_i$

b) Thm (i)  $\sum_{H_i \rightarrow X} a_i \neq 0 \Rightarrow H^p(A, a) = 0 \quad \forall p < l$   
 (suffices  $\mathcal{R}_x$  is irred.) if empty set red  
 $l = \#$  of odd char.  
 (ii)  $\mathcal{R}$  is central  $\sum_{H_i \rightarrow X} a_i \neq 0 \quad \forall x \in L \setminus \{0\}$  = B-invariant  
 $\Rightarrow H^p(A, a) \neq 0 \quad p < l-1 \quad \dim H^{l-1} = \dim H^l = |\chi(\mathcal{M}(d, \mathcal{R}))|$

(67)

Def.  $\mathbb{I} \rightarrow \Sigma_d^p = \{a \in A_1 \mid \dim H^p(A, a) \geq d\}$

resonance variety

PrZ Prove that  $R_d^p$  is affine subvar. of  $A_1$  ( $\mathbb{Z} \in \mathbb{N}$ ).

④ Propagation of cohom

Thm If  $H^p(A, a) \neq 0$  then  $H^q(A, a) \neq 0$  for  $p \leq q \leq \ell$ . (control, essential)