Lecture 11

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① A OS.

For Affine

$$0 \to A_0 \xrightarrow{\cdot a} A_1 \xrightarrow{\cdot a} \cdots \xrightarrow{\cdot a} A_\ell \to 0$$

$$I = \left\langle de_S, e_T \;\middle|\; \begin{array}{l} S = \text{circuit} \\ \bigcap_{H \in T} H = \emptyset \end{array} \right\rangle$$

Take $a \in A_1 = E_1$; $a^2 = 0$

$$\boxed{H^*(A, a) = ?}$$

$$A = \langle e_1, \dots, e_n \rangle$$
$$= \left\langle \left[\frac{d\alpha_1}{\alpha_1}\right], \dots, \left[\frac{d\alpha_n}{\alpha_n}\right] \right\rangle$$

② Motivations 1) $a \in E_1$, a is regular on A if $H^*(A, a) = 0$

otherwise a is singular

2) Relations to local Coefficients.

$$\begin{array}{ccccccccc} 0 \to & \Omega^0 & \xrightarrow{d + w_a \wedge} & \Omega^1 & \xrightarrow{d + w_a \wedge} \cdots & \xrightarrow{d + w_a \wedge} & \Omega^\ell & \xrightarrow{d + w_a \wedge} & 0 \\ & \cup & & \cup & \cup & & \cup & & \\ w \in \Omega^1 \quad 0 \to & A^0 & \longrightarrow & A^1 \to & A^2 \to & \cdots \longrightarrow & A^\ell & & \\ & & & \Psi & & & & & \\ & & & a & & & & & \end{array}$$

functions

$$\sum c_{i_1 \dots i_p} dx_{i_1} \wedge \cdots \wedge dx_{i_p}$$

skew comm. rule

under the embedding

$$a = w_a$$

$$= \sum a_i e_i = \sum a_i \frac{d\alpha_i}{\alpha_i}$$

$$(d + w_a \wedge) \frac{d\alpha}{\alpha} = w_a \wedge \frac{d\alpha_i}{\alpha_i} = a \cdot e_i$$

$$(d + w_a \wedge) w_b = a \cdot b$$

$b \in A$

Define; $\mathcal{A}$ = Boolean $\Rightarrow$ $\overset{\circ}{i}$ is quasi-iso.

if $a_i \notin \mathbb{N}$ $\quad\Rightarrow$ $\overset{\circ}{i}{}^*$ is iso.

doesn't make sense

Thm [E.S.V.] + [STV']

(Proj. Arr.) $\overset{\circ}{i}$ is quasi-iso if

$\sum_{H_i \supset X} a_i \notin \mathbb{N}$ $\quad \forall X \in L$ $\quad\Rightarrow$ $\mathcal{A}_X$ is irred.

3) "Generic" $a \in A_1$ ⇝ (vanishing)

a) Central, $\quad \sum_{i=1}^{n} a_i \neq 0 \Rightarrow H^*(A,a) = 0$

$$\begin{tikzcd}
A_{p-1} \arrow[r, "a"] & A_p \arrow[r, "a"] \arrow[dl, dashed, "h"] & A_{p+1} \arrow[dl, dashed, "h"] \\
A_{p-1} \arrow[r, "a"] & A_p \arrow[r, "a"] & A_{p+1}
\end{tikzcd}$$

If $ha + ah = 1$ then $H^*(A,a) = 0$.

Here $h = d = \sum (-1)^{j-1} e_{S_j}$

Pr 1) Well-defined 2) Prove $d \cdot a + a \cdot d = \sum_{i=1}^{n} a_i$

b) Thm (i) $\sum_{H_i \supset X} a_i \neq 0$ $\;\forall X \in L$ $\Rightarrow H^p(A,a) = 0$ $\quad \forall p < \ell$

(suffices $\mathcal{A}_X$ is irred.)

(ii) $\mathcal{A}$ is central $\sum_{H_i \supset X} a_i \neq 0$ $\;\forall X \in L \setminus \{0\}$

if complexified real $= \#$ of bdd chambers

$= \beta$-invariant

$\Rightarrow H^p(A,a) = 0$ $\quad p < \ell - 1$ $\quad \dim H^{\ell-1} = \dim H^{\ell} = |\chi(M(d\mathcal{A}))|$

Def. If $R^p_d = \{a \in A_1 \mid \dim H^p(A,a) \geq d\}$

resonance variety

Pr 2 Prove that R^p_d is affine subvar. of A_1 $(\cong \mathbb{C}^n)$.

(4) Propagation of cohom.

Thm If $H^p(A,a) \neq 0$ then $H^q(A,a) \neq 0$ for $p \leq q \leq \ell$. (central, essential)