

LGS Formula

\mathcal{R} central ext. $G = \pi_1(M)$

$G = G_1 \supseteq G_2 \supseteq \dots \supseteq G_n \supseteq \dots$ L.C.S.

$$G_i = [G_{i-1}, G]$$

$$\varphi_i(\mathcal{R}) = \text{rank } G_i / G_{i+1}$$

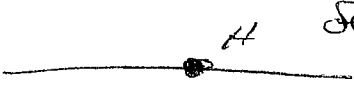
G_i / G_{i+1} abelian, f.g.
not necessarily
torsion free

$$P_M(t) = \text{Poin}(\mathcal{R}, t)$$

$$(\mathbb{Q} \oplus \mathbb{Q})$$

LCS formula

$$\prod_{k \geq 1} (1 - t^k)^{\varphi_k(\mathcal{R})} = P_M(-t)$$

First^o ex:  $\mathcal{R} = \{0\}$ $G = \mathbb{Z}$

$$\varphi_1 = 1 \quad \varphi_2 = 0 = \varphi_k \quad k \geq 2$$

$$(1-t) = 1-t$$

First Example

(107)

Kahno: $\mathcal{A} = \mathcal{A}_g$ - Braid arr.

$$\Rightarrow \prod_{k \geq 1} (1-t^k)^{\varphi_k} = (1-t)(1-2t)(1-3t) \cdots (1-(g-1)t)$$

pf: rational homotopy theory / homomorphisms Lie algebras

\mathcal{A}_g is supersolvable / fiber-type w/ exponents $1, 2, \dots, g-1$

$$\begin{array}{ccccccc} M(\mathcal{A}_g) & \longrightarrow & M(\mathcal{A}_{g-1}) & \longrightarrow & M(\mathcal{A}_{g-2}) & \longrightarrow & \cdots \longrightarrow M(\mathbb{C}) \\ \uparrow & & \uparrow & & \uparrow & & \\ \mathbb{C} \setminus (g-1 \text{ pts}) & \longrightarrow & \mathbb{C} \setminus (g-2 \text{ pts}) & \longrightarrow & \cdots & & \end{array}$$

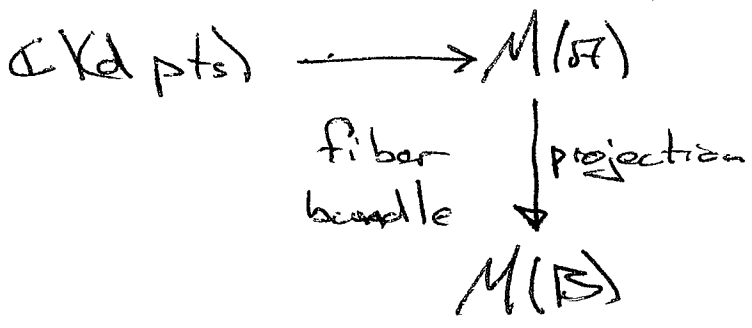
\mathbb{C}^*
 \parallel

$$\pi_1(\mathbb{C} \setminus (d \text{ pts})) \cong F_d$$

Witt's formula

$$\prod_{k \geq 1} (1-t^k)^{\varphi_k(F_d)} = 1-dt$$

$$\varphi_k(F_d) = \sum_{r|k} \frac{1}{k} \mu\left(\frac{k}{r}\right) d^r$$



fiber type \Downarrow
 $(M(\mathbb{B}) \text{ is aspherical})$

long exact htpy seq.

$$(\mathbb{B}) \rightarrow \pi_1(\mathbb{C}\text{-pts}) \rightarrow \pi_1(M(\mathbb{C}^d)) \rightarrow \pi_1(M(\mathbb{B})) \rightarrow 1$$

So we have

$$1 \rightarrow \overset{A}{F_d} \rightarrow \overset{B}{\pi_1(M(\mathbb{C}^d))} \xrightarrow{\cong} \overset{C}{\pi_1(M(\mathbb{B}))} \rightarrow 1$$

Ingredients: (i) \exists a section
 (ii) action of $\pi_1(M(\mathbb{B}))$ on F_d is trivial on $F_d/[F_d, F_d]$

$$1 \rightarrow A \rightarrow B \rightarrow C \rightarrow 1$$

hard work $\Rightarrow 1 \rightarrow A_n \rightarrow B_n \rightarrow C_n \rightarrow 1$

$\Rightarrow 1 \rightarrow \frac{A_n}{A_{n+1}} \rightarrow \frac{B_n}{B_{n+1}} \rightarrow \frac{C_n}{C_{n+1}} \rightarrow 1$
 from free SP

Conclusion (by induction)

B_n/B_{n+1} is free abelian and

$$\text{rk}(B_n/B_{n+1}) = \text{rk}(C_n/C_{n+1}) + \text{rk}(A_n/A_{n+1})$$

For \mathcal{Z} fiber type w/ ~~exp~~ fibers $C-d_i$ pts $i=1, \dots, l$

\Rightarrow ① $P_n(t) = \prod_{i=1}^l (1+d_i t)$

② $\phi_k(G) = \sum_{i=1}^l \phi_k(F_{d_i}) \rightarrow \prod_{k \geq 1} (1-t^k)^{\phi_k(G)}$

③ C_n/C_{n+1} is free Abelian

$$\prod_{i=1}^l \prod_{k \geq 1} (1-t^k)^{\phi_k(F_{d_i})} = \prod_{k \geq 1} (1-t^k)^{\sum \phi_k(F_{d_i})}$$

|| With

$$\prod_{i=1}^l (1-d_i t) = P_n(-t)$$

Thus If \mathcal{Z} is fiber type then this LCS formula holds.