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Special pencils of plane curves, multinets, and resonance varieties

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ABSTRACT

We study three properties of a finite set $\mathcal{A}$ of lines in a complex projective plane. Here are their (unprecise) definitions:

(i) The set $\mathcal{A}$ can be partitioned in several blocks (at least three) so that the pencil of curves, containing the unions of lines in two of the blocks as two fibers, contains also the union for every other block. We also require the fibers to stay connected after blowups at all the base points.

(ii) The set $\mathcal{A}$ supports a multinet, that is from combinatorics point of view a deformation of a net with certain points glued together. A little more precisely, $\mathcal{A}$ again can be partitioned in several blocks so that at each intersection point of lines from different blocks each block is represented by the same number of lines counted with some integer multiplicities.

(iii) The set $\mathcal{A}$ supports a resonance component. This means that there is an element d in $H^1(M)$ supported on all the lines (where M is the complement of $\mathcal{A}$) such that the complex $H^*(M)$ with the multiplication by d as the differential has non-vanishing first cohomology.

Our main result says that all these properties are equivalent to each other.