"Arrangements in Boston"
A Conference on Hyperplane Arrangements

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Northeastern University
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Polytopal bundle theory

Laura Anderson
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A polytopal bundle is a combinatorial analog to a topological sphere bundle, in which the base space is replaced by a poset and the fibers are replaced by combinatorial types of convex polytopes. The combinatorial analog to continuity is expressed in terms of polytopal subdivision. We have recently proven that the category of polytopal bundles is equivalent to the category of piecewise-linear sphere bundles. As immediate consequences we get new topological results on various categories of combinatorial types of polytopes.

Joint work with Nikolai Mnëv (Steklov Institute)

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On loop spaces of configuration spaces, and braid-like groups

Fred Cohen
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The main topic in this lecture is the structure of loop spaces of classical configuration spaces for k ordered distinct points in a manifold M.

(1) The singular homology of these spaces admit structures which are frequently extensions of the "universal Yang-Baxter Lie algebra" and which depend on the underlying properties of the tangent bundle for M.

(2) There are natural associated groups to these Lie algebras which satisfy properties analogous to those of Artin's braid groups, and are gotten by assembling the images of the classical Hurewicz homomorphism for high dimensional homotopy groups. Furthermore, these groups admit a topological interpretation of "braiding" certain subspaces of a manifold via a "time" parameter.

(3) Some parallels with invariants of pure braids together with their homotopy theoretic interpretations are given. For example, one of the Lie algebras above also arises as the Lie algebra attached to the descending central series for Artin's pure braid group, and thus via Vassiliev invariants of pure braids. There are analogous Lie algebras "with symmetries" constructed by M. Xicoténcatl for "orbit configuration spaces".

Joint work with Sam Gitler

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Four planes

Henry Crapo
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With an eye to applications to the mechanics of bar and joint frameworks, and with a desire to find some unity in several parallel approaches to classification of subspace configurations, we practice on the 49 distinct figures of four planes in 4-dimensional space.

Joint work with Janos Baracs

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Tessellations of real moduli spaces

Satyan Devadoss
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We look at the real points of the moduli space of punctured Riemann spheres. This space is naturally tiled by Stasheff associahedra, which can be described as polygons with non-intersecting diagonals. We look at the cellular decomposition of the moduli space and describe its structure from the viewpoint of real blow-ups of hyperplane braid arrangements.

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Quadratic algebras and line-closed matroids

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Let G be a matroid and $A(G)$ the associated Orlik-Solomon algebra. Recall that A is *quadratic* if and only if the relation ideal is generated by elements of degree two. A set S of points is line-closed in G if, for every $x, y \in S$, the entire line spanned by x and y is contained in S. The matroid G is *line-closed* if any line-closed set is closed.

In this talk we describe work in progress toward a proof of the following conjecture: $A(G)$ is quadratic if and only if G is line-closed. The proof is more interesting than the conjecture. Fix a linear order of the points of G. We construct a natural generalization of the well-known **nbc** basis of $A(G)$, which contains the usual basis, and coincides with it if and only if G is line-closed. We prove that these 2-**nbc** bases are linearly independent in the quadratic closure of $A(G)$, yielding one of the implications in the conjecture.

The number of 2-**nbc** bases changes when the linear order is changed, a reflection of the fact that line-closure does not satisfy the Steinitz exchange axiom. We give some indications that for some special linear orders, the 2-**nbc** bases give bases for the quadratic closure of $A(G)$, yielding a proof of the conjecture and a combinatorial calculation of the "global invariant" $\phi_3(A)$ introduced in our earlier work. We also describe how this idea can be extended to give a sequence of combinatorial/algebraic/topological invariants of matroids, OS algebras, or hyperplane arrangements.

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On the cohomology of orbit configuration spaces of spheres

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We consider spaces of point configurations on spheres, where points are required to be pairwise distinct and non-antipodal. These spaces are basic instances of so-called orbit configuration spaces that have recently attracted interest in the study of equivariant function spaces.

We focus on determining the integer cohomology algebras of the orbit configuration spaces of spheres described above. The topological theory of hyperplane arrangements and of subspace arrangements turned out to be essential for determining cohomology of classical ordered configuration spaces of spheres, as we have shown in earlier work. We review these results and discuss the extension of our approach to orbit configuration spaces. Again, complements of subspace arrangements figure prominently, though in a less direct way. We are left with an interesting investigation on the borderline of linear and non-linear structure.

Joint work with Günter M. Ziegler (TU Berlin)

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Hyperplane arrangement and semisimple orbits

Jason Fulman
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We describe conjectures about counting semisimple orbits of the adjoint action of a finite group of Lie type on its Lie algebra. By a theorem of Steinberg the total number of such orbits is q to the rank of the group, but their description remains a difficult problem. Grouping these orbits by "genus" suggests a formula involving counting solutions of equations which arose in work of Sommers on affine flag manifolds. These expressions have a quite different flavor from work of Lehrer in type A. Grouping these orbits by the conjugacy class of the Weyl group which they map to suggests formula involving characteristic polynomials of sublattices of the root hyperplane intersection lattice. These ideas tie in with a natural notion of riffle shuffling for real hyperplane arrangements.

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Massey products as tree-level parts of 3-manifold invariants

Stavros Garoufalidis
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We will provide relations among the following subjects, as well as concrete statements and proofs:

1. The tree-level part of a theory of finite type invariants of 3-manifolds (in general this theory has graphs of arbitrary number of loops).
2. Massey products.
3. The Johnson homomorphism.
4. Deformation quantization of surfaces.
5. Graph-cohomology.

Our proofs are a mix of graph cohomology, geometric topology and a bit of surgery theory in the classical dimension.

Joint work with Jerome Levine (Brandeis University)

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Hypersolvable arrangements

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We introduce the class of hypersolvable arrangements which contains both the fiber-type ones and the generic ones.

This class extends and refines various results concerning the topology of the complement, in its interplay with the combinatorics. For example, the $K(\pi, 1)$-property is combinatorial in the hypersolvable class along with some other properties conjectured to be related to asphericity.

Joint work with Stefan Papadima

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On the Gauss-Manin connection and hypergeometric representations of the pure braid group

Herbert Kanarek
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We study local systems arising from flat line bundles over topologically trivial families $U \to S$ of hyperplane complements in $\mathbb{P}^n$. Imposing some genericity condition on the monodromy, one knows that fiberwise the cohomology of the local system is concentrated in the middle dimension and is computed by the Aomoto complex, a subcomplex of global differential forms on a good compactification $\pi : X \to S$ with logarithmic poles along $D' = X \setminus U$.

The families $\mathcal{A}'$ considered are obtained by fixing a configuration $\mathcal{A}$ of hyperplanes and moving one additional hyperplane. The line bundle is the structure sheaf, endowed with the connection $d_{rel} + \omega$, for a logarithmic relative differential form ω. In this situation we construct the Gauß-Manin connection ∇ on $R^n\pi_*(\Omega^\bullet_{X/S}(\log D'), d_{rel} + \omega)$. We show that these sheaves are free. Using the combinatorics of $\mathcal{A}'$ we give a basis for these sheaves and an algorithm to express the connection ∇ in this basis.

These results can be seen as a generalization of the hypergeometric functions.

We apply this method for the case when $\mathcal{A}$ is a configuration of $n + 2$ hyperplanes in general position in $\mathbb{P}^n$. As discriminant of $\mathcal{A}$ we have the braid configuration. Calculating the monodromy of this system we obtain hypergeometric representations of the pure braid group.

We illustrate the method with some examples.

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On representation varieties of Artin groups, projective arrangements and the fundamental groups of smooth complex algebraic varieties

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We prove that for any affine variety S defined over $\mathbf{Q}$ there exists an Artin group G such that a Zariski open subset U of S is biregular isomorphic to a Zariski open subset of the character variety $Hom(G, PO(3))//PO(3)$. The subset U contains all real points of S. As an application we construct new examples of finitely-presented (Artin) groups which are not fundamental groups of smooth complex algebraic varieties. The proof is based on a scheme-theoretic version of Mnev's universality theorem and on the following generalization of Morgan's test for the fundamental groups of smooth algebraic varieties:

Suppose M is a smooth connected complex algebraic variety with the fundamental group G, L is a reductive algebraic Lie group and $f : G \to L$ is a representation with finite

image. Then the germ $(Hom(G, L), f)$ is analytically isomorphic to a quasi-homogeneous cone with generators of weights 1 and 2 and relations of weights $2, 3$ and 4. In the case there is a local cross-section through f to $Ad(L)$-orbits, then the same conclusion is valid for the quotient germ $(Hom(G, L)//L, [f])$.

Joint work with John J. Millson (University of Maryland)

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The mixed Hodge structure on the fundamental group of a complement of hyperplanes

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The complement of an arrangement of hyperplanes is a good example of the mixed Hodge structure on the fundamental group of an algebraic variety. We compute its isomorphism class using iterated integrals and then get the cross ratio of a arrangement that is a combinatorial and projective invariant. So cross ratio equivalent arrangements are isomorphic. Moreover, an isomorphism of polarized mixed Hodge structures on the fundamental group induces cross ratio equivalent arrangements.

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Braid group of the torus and elliptic KZ system

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We begin with a description of Vassiliev invariants of braids in terms of the bar complex of the Orlik-Solomon algebra of a configuration space and discuss its generalization to the braid group of the torus based on the elliptic KZ system.

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How to calculate characteristic polynomials if you must

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Characteristic polynomials (CP) are difficult to calculate. There are several identities which help. We shall discuss two lesser known identities. One is due to Henry Crapo. This expresses the CP of a submatroid as a sum of CP's of contractions of the matroid. The other is an explicit formula for the "correction term" in Stanley's modular factorization identity when the flat is not modular. We shall give several applications. One of them is a way to construct many arrangements whose CP's factor completely over the integers but are not free.

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Sums and integrals over polytopes and quantum invariants

Ruth Lawrence
University of Michigan (Ann Arbor) and Hebrew University (Jerusalem)

We discuss recent results on the structure of quantum 3-manifold invariants in the context of properties of sums over integer points contained in a polytope.

The Witten-Reshetikhin-Turaev quantum invariants $Z_K(M)$ of 3-manifolds, M, are complex number invariants dependent on the choice of a Lie algebra, and of a root of unity, q, of order K. For rational homology spheres, it is known that the collection of invariants of a fixed manifold as K varies has additional structure:

- It is a family of algebraic integers. (Murakami)
- It is connected to the Ohtsuki power series invariant via K-adic convergence for prime K. (Rozansky)
- For some simple manifolds, it has a simple holomorphic extension. (Jeffrey; L.; L.-Rozansky; L.-Zagier)

We will see that these properties are related to the general structure of a sum of the form

$$\sum_{\mathbf{x}\in KR\cap Z^n} f(q,\mathbf{x})$$

where R is a rational polytope and f is periodic in $\mathbf{x}$ of period K (for $q^K = 1$). In particular we discuss when such a sum can be represented by an integral of f over a region (or finite collection of regions) whose dependence on K is limited to its congruence class mod P, some fixed P.

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First order deformations for rank one local systems with non vanishing cohomology

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We describe a tangent cone to the variety of rank one local systems on a finite CW-complex having the dimension of k-dimensional cohomology exceeding m. This cone is identified with the space of certain complexes of abelian groups with differential induced by the cup product. In the case when the CW-complex is a quasiprojective complex algebraic variety the space of such complexes is a union of linear spaces. In particular, for an arrangement of hyperplanes, the set of 1-forms such that the corresponding Aomoto complex has k-dimensional betti number exceeding m is a union of linear space.

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Finite index subgroups of arrangement groups

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Let G be a finitely presented group, and p a prime number. A natural invariant of G is the number $N_p(G) = \#\{K \lhd G \mid [G:K] = p\}$ of index p normal subgroups. We introduce a series of numerical invariants of G by counting the index p normal subgroups K of G according to

- their first Betti number:

$$\beta_{p,d}(G) = \#\{K \mid b_1(K) = b_1(G) + (p-1)d\};$$

- the q-torsion of their abelianization, for a prime $q \neq p$:

$$\gamma_{p,q,d}(G) = \#\{K \mid \dim_{\mathbb{Z}_q} \operatorname{Tors} H_1(K) \otimes \mathbb{Z}_q = (p-1)d + \dim_{\mathbb{Z}_q} \operatorname{Tors} H_1(G) \otimes \mathbb{Z}_q\};$$

- the p-torsion of their abelianization:

$$\nu_{p,d}(G) = \#\{K \mid \dim_{\mathbb{Z}_p} \operatorname{Tors} H_1(K) \otimes \mathbb{Z}_p = d\}.$$

In this talk we discuss the above invariants when G is either the fundamental group of the complement of an arrangement, or a nilpotent quotient of such a group. We consider both complex line arrangements in $\mathbb{C}^2$ and real plane arrangements in $\mathbb{R}^4$.

The central idea we explore is that the numbers $N_p(G)$, $\beta_{p,d}(G)$, $\gamma_{p,q,d}(G)$, and $\nu_{p,d}(G)$ are determined by certain algebraic varieties (over fields of appropriate characteristic) associated to the group G: the characteristic varieties and the resonance varieties.

Joint work with Alexander I. Suciu (Northeastern University)

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Geometry of sextics and their dual curves

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In this talk, we will explain some interesting geometry of cuspidal sextics. We consider the moduli space $\mathcal{M}$ of sextics with six cusps and three nodes. It is self-dual by the dual curve operation. The submoduli of curves of torus-type is denoted by $\mathcal{M}_{torus}$. We will show that:

Theorem 1.

1. $\mathcal{M}$ is self-dual by the dual curve operation. Furthermore, the dual operation preserves curves of torus-type and non-torus-type. For $C \in \mathcal{M}$, C is in $\mathcal{M}_{torus}$ iff $\Delta_C(t) = t^2 - t + 1$.
2. Let $\widehat{\mathcal{M}}_{torus}$ be the connected component of $\mathcal{M}(24\beta_{2,3} + 24\beta_{2,2}; 12)$ which contains a curve of torus-type. Then, $\widehat{\mathcal{M}}$ consists of (maybe not all) quasi-torus curves and $\widehat{\mathcal{M}}_{torus}$ is also invariant under the $\star$ operation. Their Alexander polynomials are given by $t^2 - t + 1$.
3. There exists a canonical morphism $\psi : \mathcal{M}_{torus} \to \widehat{\mathcal{M}}_{torus}$ and an involution $\iota : \mathcal{M}_{torus} \to \mathcal{M}_{torus}$ ($\iota \neq \star$) such that the diagram commutes:

$$\begin{array}{ccc} \mathcal{M}_{torus} & \xrightarrow{\iota} & \mathcal{M}_{torus} \\ \downarrow \psi & & \downarrow \psi \\ \widehat{\mathcal{M}}_{torus} & \xrightarrow{\star} & \widehat{\mathcal{M}}_{torus} \end{array}$$

Theorem 2. (Self-dual three $(3,4)$-cuspidal sextics).

There exists a unique self-dual sextic with three $(3,4)$-cusps. It is of torus-type and is given by $f := f_2^3 + 54 f_3^2$, where $f_2 := y^2 - 2x + x^2, \quad f_3 := (y^2 - x^2)(x-1)$.

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Arrangements and local systems

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We use stratified Morse theory to build a complex to compute the local system cohomology of the complement of a hyperplane arrangement.

Theorem: The linearization of this complex is the Orlik-Solomon algebra with the connection operator.

Using this result, we obtain lower bounds for the local system Betti numbers in terms of those of the Orlik-Solomon algebra, recovering a result of Libgober and Yuzvinsky. We also establish the relationship between the cohomology support loci of the complement and the resonance varieties of the Orlik-Solomon algebra for any arrangement, and show that the latter are unions of subspace arrangements in general, resolving a conjecture of Falk.

For certain local systems, our results provide new combinatorial upper bounds on the local system Betti numbers. These upper bounds enable us to prove that in nonresonant systems the cohomology is concentrated in the top dimension, without using resolution of singularities.

Joint work with Daniel C. Cohen (Louisiana State University)

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The universal finite-type invariant for braids, with integer coefficients

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We give a simple explicit construction of the universal finite-type invariant for braids. Our construction works well for arbitrary coefficients. A key ingredient is provided by the properties of fundamental groups of fiber-type arrangements.

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On surface braid groups

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Throughout the lecture, M will denote a compact surface, not necessarily oriented and possibly with boundary. Choose a finite collection $P = \{p_1, \ldots, p_m\}$ of m points (called punctures) in the interior of M. Define a braid with m strings on M based at P to be a collection $b = \{b_1, \ldots, b_m\}$ of m paths, $b_i : [0,1] \to M$ such that:

(1) $b_i(0) = p_i$ and $b_i(1)$ in P for all i in $\{1, \ldots, m\}$;

(2) $b_i(t) \neq b_j(t)$ for i, j in $\{1, \ldots, m\}$, $i \neq j$, and t in $[0,1]$.

There is a natural notion of homotopy of braids. The braid group on m strings based at P is the group $B(M,P)$ of homotopy classes of braids based at P. The group operation is concatenation of braids, generalizing the construction of the fundamental group.

After explaining the link between braid groups and configuration spaces, we will talk about torsion, centers, presentations, and other combinatorial aspects of these groups.

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Hyperbolic line arrangements, baskets, arborescent Seifert surfaces, generalized positive braids, and unfoldings

Lee Rudolph
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Let $\mathcal{L}$ be an arrangement of lines $L_1, \ldots, L_m$ in the (real) hyperbolic plane which is transverse at infinity (i.e., for $i \neq j$, L_i and L_j don't share an ideal endpoint). To every weighting $w : \{1, \ldots, m\} \to \mathbf{Z}$ there corresponds a basket $S(\mathcal{L}, w)$ in S^3 (i.e., a Seifert surface formed by plumbing unknotted twisted annuli to a 2-disk); conversely, every basket is isotopic to $S(\mathcal{L}, w)$ for some $\mathcal{L}$ (which may, but need not, be taken to also be transverse in the finite plane, i.e., to have no three concurrent lines) and some w. For example, the arborescent Seifert surface corresponding, in the usual way, to an evenly weighted planar tree $(\mathcal{T}, w)$ is a basket isotopic to $S(\mathcal{L}, -w/2)$ where $\mathcal{L}$ is a line-tree dual to $\mathcal{T}$ in a manner strictly analogous to the duality between "plumbing diagrams" and "resolution trees" in the theory of graph manifolds and resolution of surface singularities.

The surface $S(\mathcal{L}, w)$ is a fiber surface if and only if the weighting w takes values in $\{1, -1\}$, in which case $S(\mathcal{L}, w)$ is a Hopf-plumbed basket. In particular, for any $\mathcal{L}$, the Hopf-plumbed surface $S(\mathcal{L}, -1)$ corresponding to the constant weighting -1 is a quasipositive Hopf-plumbed basket with an alternative representation as the braided surface corresponding to an appropriate positive word in a set of generators $\sigma_{\mathbf{e}}$ of some braid group B_n, $n < m$, corresponding to the edges $\mathbf{e}$ of an espaliered tree with vertices $\{1, \ldots, n\}$. For example, for $n = 1$ (where the espaliered tree has a single edge) and $k > 0$, the word σ_1^k corresponds to a torus knot or link $O\{2, k\}$ and to a hyperbolic line arrangement of type A_{k-1}.

In the quasipositive fibered case, at least when $\mathcal{L}$ is actually a euclidean line arrangement (and presumably in general, once appropriate definitions are supplied), not only does the complex line arrangement in $\mathbf{C}^2$ obtained by complexifying $\mathcal{L}$ intersect every sufficiently large S^3 in a fibered link of type $\partial S(\mathcal{L}, -1)$, but in fact the map $\mathbf{C}^2 \to \mathbf{C}$ obtained by treating $\mathcal{L}$ as a divide (in a sense slightly generalizing that of A'Campo) is an unfolding (in the sense of Neumann and Rudolph) of $\partial S(\mathcal{L}, -1)$. The unfolding is complete if and only if $\mathcal{L}$ is transverse in the finite plane.

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Cohomology of Artin groups over local coefficients

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For all Artin groups associated to finite, irreducible Coxeter groups, we give a complete description of the cohomology with coefficients in naturally associated local systems over $\mathbb{Q}[q, q^{-1}]$.

The answer is particularly interesting and neat in the case of classical braid groups, where the cohomology table has quite "visible" arithmetical properties.

The proof uses previous constructions and spectral sequence-like arguments.

Joint work with C. De Concini and C. Procesi

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The module of logarithmic p-forms of a locally free arrangement

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For an essential, central arrangement $\mathcal{A} \subseteq V \simeq K^{n+1}$ (char $K = 0$), let J be the Jacobian ideal of the defining polynomial of $\mathcal{A}$. We show that $\Omega^1(\mathcal{A})$ (the module of logarithmic one forms with pole along $\mathcal{A}$) gives rise to a locally free sheaf on $\mathbf{P}^n$ iff the sheaf $\mathcal{E}xt^i_{\mathcal{O}_{\mathbf{P}^n}}(\mathcal{O}_{\mathbf{P}^n}/\mathcal{J}, \mathcal{O}_{\mathbf{P}^n})$ is zero, for all $i > 2$, and prove that this condition holds if and only if for all $X \in L_{\mathcal{A}}$ with rank $X < \dim V$, the subarrangement $\mathcal{A}_X$ is free. $\Omega^1(\mathcal{A})$ has a direct sum decomposition as $\Omega^1_0 \oplus S(1)$; we prove that the sheaf defined by $\Omega^1(\mathcal{A})$ is locally free if and only if the module of logarithmic p-forms $\Omega^p(\mathcal{A})$ and the module $\Lambda^p(\Omega^1(\mathcal{A}))$ have the same sheafification (so as modules, have isomorphic saturations). In this situation, we show that if either $\Omega^1(\mathcal{A})$ or its dual has projective dimension at most one, then (writing c_t for the Chern polynomial) $(1+t) \cdot c_t(\widetilde{\Omega}^1_0) = \Pi(\mathcal{A}, t)$. The class of locally free arrangements for which this condition holds includes free arrangements, arrangements in $\mathbf{P}^2$, and generic arrangements, and this inclusion is proper.

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Hyperplanes arrangements and graph orientations

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Let G be a graph on vertices $x_1, \ldots, x_n$. For each edge (x_i, x_j) in G, the linear equation $x_i = x_j$ defines a hyperplane in R^n. Call the arrangement of these hyperplanes H_G. It is known that the regions defined by H_G are in one-to-one correspondence with acycylic orientations of G. (An orientation of G is an assignment of directional arrows to the edges of G. It is acyclic if no circle (i.e. circuit) in G has cohernt directional arrows.) This provides a nice graph-theoretical description of the regions of an arrangement of hyperplanes. It is also known that this graph-theoretical description is sufficient to represent all orientations of the matroid associated with H_G.

Consider a graph G along with a labeling f of the directed edges of G with nonzero real numbers in which $f(x_i, x_j) = 1/f(x_j, x_i)$. For each edge (x_i, x_j) in G, the linear equation $x_i = f(x_i, x_j)x_j$ defines a hyperplane in R^n, call the arrangement of these hyperplanes $H_{G,f}$. We will give a graph-theoretical description of the regions defined by $H_{G,f}$. We will also show that this graph-theoretical description, in a number of cases but not in general, suffices to describe all orientations of the matroid associated with $H_{G,f}$.

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Hyperplane arrangements and interval orders

Richard Stanley
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Let P be a set of closed intervals $[a, b]$ on the real line. Define $[a, b] < [c, d]$ if $b < c$. Then P becomes a partially ordered set called an *interval order*. There is a close connection between interval orders and the theory of hyperplane arrangements. For instance, the nonisomorphic labelled interval orders that can arise from n labelled intervals of length one are in one-to-one correspondence with the regions of the arrangement $x_i - x_j = -1, 1$, for $1 \leq i < j \leq n$, in $\mathbf{R}^n$. We will survey the basic connections between interval orders and hyperplane arrangements and give some applications to the enumeration of interval orders.

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Iterated residues and Bernoulli sums

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We review recent work on computation of multi-dimensional sums using iterated residues. Consider the ring of rational functions with poles along the elements of some real central hyperplane arrangement. The goal is to study invariantly defined functionals on this ring. It turns out that there is a well defined notion of the "constant" term of a function in this ring, which leads to a more conceptual understanding of broken-circuit bases via iterated residues.

If one fixes a compatible lattice of full rank, then one can consider the sum of the values of a rational function on the regular part of the lattice (a Bernoulli sum). In the multidimensional context, such sums first appeared in the work of Witten on 2-dimensional gauge theory. In a certain sense, this summation is again an invariant functional, which is closely related to the constant term. This allows one to write explicit formulas for these Bernoulli sums.

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Milnor fiber complexes for Shephard groups

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The symmetry group of a regular polytope is a finite Coxeter group. The intersection of the unit sphere with the reflecting hyperplanes of the corresponding Coxeter arrangement induces a simplicial triangulation of the sphere, called the Coxeter complex. A Shephard group G is the symmetry group of a regular complex polytope. Orlik has shown that there can be associated to G a real simplicial complex which possesses many properties analogous to the Coxeter complex. Let f_1 be the G-invariant polynomial of minimal positive degree, and let $F = f_1^{-1}(1)$ be its Milnor fiber. Orlik showed that there is a complex Γ which is an equivariant strong deformation retract of F, is G-stable, is stratified by the associated reflection arrangement, and which satisfies specific cell-counting formulas. His proof is existential; it does not give an explicit method of constructing the complex, though Orlik and Solomon did explicitly construct complexes for one infinite family of Shephard groups. Here we describe the construction of complexes for the remaining 15 "exceptional" Shephard groups.

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Flat connections arising from a family of arrangements

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A combinatorially stable family of arrangements determines a flat connection on the moduli space of the family. When the family is universal, the connection is completely determined by the combinatorial type of the arrangements. We study the exlplicit form of the connection matrices presented with respect to β nbc-bases.

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Quantization of geometry associated with the qKZ equation

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Cohomology groups of the complement to an arrangement with coefficients in a local system were a subject of recent studies. The quantum Kniznik-Zamolodchikov difference equation suggests a discrete version of this topic, in which powers of linear functions are replaced by gamma functions, such geometric objects like cycles are replaced by exponential functions and all basic relations remain preserved.

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Orbit configuration spaces and equivariant loop spaces

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Given a manifold M with a G-action, we analyze an equivariant version of the ordinary configuration spaces of Fadell and Neuwirth. After deriving their basic properties, we look at the homology and loop space homology of some examples, which turn out to be hyperplane arrangements. An interesting fact is the appearence of equivariant "infinitesimal pure braid relations" and an extra relation in the description of the latter.

Secondly, we show how to use theses spaces to construct a combinatorial model for the space of equivariant loops of a space X, which we use it to give an splitting of $(\Omega^n\Sigma^n X)^{\mathbb{Z}_2}$ for a well pointed $\mathbb{Z}_2$-space X. These methods can also be adapted to study $(\Omega^n\Sigma^n X)^G$ for a finite group G.

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Resolutions of ideals of polynomial and exterior algebras

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We discuss the class of homogeneous ideals of a polynomial ring whose Taylor complex is a resolution. For ideals of this class (that includes monomial ideals) we discuss how to find their Betti numbers and multiplication in Tor-ring combinatorially. We give a combinatorial construction of a minimal resolution. Some initial results for ideals of exterior algebras and applications to arrangements are also discussed.

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Supersolvable matroids of signed and biased graphs

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A signed graph is a graph in which each edge is labelled $+$ or $-$. A polygon (i.e., circuit) is called "balanced" when the product of its edge signs is positive. If we want to understand the linear dependence structure of a subset S of the dual hyperplane arrangement of a classical root system, — let's say, of $C_n^* = \{x_i = x_j, x_i = -x_j\}$ in R^n — we can do so through a matroid $G(\Sigma)$ which is defined on the edge set of a signed graph Σ (corresponding to S) in terms of the balance of polygons in Σ. $G(\Sigma)$ faithfully represents the linear dependence properties of S. This gives us a graphical, and reasonably practical, way to calculate the matroidal characteristic polynomial of S (a polynomial that is better known to some, these days, as the Hilbert polynomial of the hyperplane arrrangement S^* dual to S) and to study such properties of S^* as its algebraic (i.e., Terao) freeness and supersolvability that imply an integral factorization of the polynomial.

A gain graph is like a signed graph but with the group of signs replaced by an arbitrary group. (When the group is finite cyclic, we can use gain graphs to represent the linear dependence structure of a subset of a complex analog of a root system.) A biased graph is a combinatorial abstraction of signed and biased graphs, much as an abstract projective geometry abstracts the projective space over a field, which is just strong enough to permit a matroid theory.

With biased graph theory it is easy to characterize which signed, gain, and biased graphs have matroids that are supersolvable, generalizing Stanley's theorem about graphic matroids. By gain graph coloring, it is easy to perform a graphical calculation of the characteristic polynomial; with such calculations and the characterization of supersolvability one can generalize to gain graphs Edelman and Reiner's description of the signed graphs that contain all possible positive nonloop edges and are supersolvable or algebraically free.

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Cohomology algebras of subspace arrangements — the combinatorial approach

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"Combinatorial Stratifications" of real and complex subspace arrangements lead to explicit cell complex models for the complements — and in favorable cases one has enough combinatorial control to derive in this setting not only the cohomology of the complement, but even the multiplicative structure of the cohomology algebra.

In this talk, I plan to review the set-up, and report about some recent "favorable cases":

- my study with Eva Maria Feichtner on the cohomology algebras of subspace arrangements with a geometric lattice of intersections,
- work by Mark de Longueville on the cohomology algebras of real and complex coordinate subspace arrangements, and
- the results by Corrado De Concini and Mario Salvetti on the cohomology of finite Coxeter groups.

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