Prof. Alexandru Suciu

MTH U565 **TOPOLOGY** **Spring 2006**

HOMEWORK 5

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1. Let X be a topological space. Suppose there are two subspaces, A and B, such that
- $X = A \cup B$
- $A \cap B \neq \emptyset$
- A and B are connected.

Show that X is connected.

2. Let $f\colon X \to Y$ be a continuous function. Suppose X is path-connected. Show that $f(X)$ is also path-connected.
As a corollary, show that path-connectedness is a topological invariant.

3. Let $\mathcal{T}$ and $\mathcal{T}'$ be two topologies on the set X. Suppose $\mathcal{T}' \supset \mathcal{T}$. What does connectedness of X under one of these topologies imply about connectedness under the other?

4. Let $\mathcal{T}$ and $\mathcal{T}'$ be two topologies on the set X. Suppose $\mathcal{T}' \supset \mathcal{T}$. What does compactness of X under one of these topologies imply about compactness under the other?

5. Let $A_1, \dots, A_n$ be compact subspaces of a space X. Show that

$$A = \bigcup_{i=1}^{n} A_i$$

is also compact.