Prof. Alexandru Suciu

MATH 4565 **TOPOLOGY** **Spring 2010**

FINAL EXAM

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1. Let $f\colon X \to Y$ be a continuous, bijective map. Recall the following theorem: If X is compact and Y is Hausdorff, then f is a homeomorphism. Show that both assumptions are necessary for the theorem to hold. That is,

(a) Provide an example where $f\colon X \to Y$ is a continuous, bijective map and X is compact, but f is not a homeomorphism.

(b) Provide an example where $f\colon X \to Y$ is a continuous, bijective map and Y is Hausdorff, but f is not a homeomorphism.

2. Let $X = \{(x, y) \in \mathbb{R}^2 \mid x \in \mathbb{Q} \text{ or } y \in \mathbb{Q}\}$ be the set of all points in the plane with at least one rational coordinate. Show that X, with the induced topology, is a path-connected space.

3. Let f and g be paths in $\mathbb{R}^2 \setminus \{0\}$. Show that f is homotopic to g.

4. Consider the unit circle $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$. Let $f\colon S^1 \to S^1$ be the map defined by $f(x, y) = (-x, y)$. What is the degree of f?

5. Let $f\colon S^1 \to S^1$ be a continuous map. Suppose $\deg(f) \neq 0$. Show that f is surjective.

6. Let X be a Hausdorff space, and let A be a retract of X. Show that A is a closed subset of X.

7. A subspace $A \subset X$ is called a *deformation retract* of X if there is a retraction $r\colon X \to A$ with the property that $i \circ r \simeq \mathrm{id}_X$. Prove the following: if A is a retract of a contractible space X, then A is a deformation retraction of X.

8. Prove the following:

(a) The open interval $(0, 1)$ is *not* a retract of the real line $\mathbb{R}$.

(b) The closed interval $[0, 1]$ is a deformation retract of the real line $\mathbb{R}$.