

QUIZ 7

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1. 7 points Let  $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 9 & -5 \end{bmatrix}$ .

(a) Find the eigenvalues of  $A$ .

(b) Find a basis for each eigenspace of  $A$ .

(c) Find a diagonal matrix  $D$  and an invertible matrix  $S$  such that  $A = S \cdot D \cdot S^{-1}$ . (You need not compute  $S^{-1}$ .)

2. 6 points A  $4 \times 4$  matrix  $A$  has eigenvalues  $\lambda_1 = -4$ ,  $\lambda_2 = -1$ ,  $\lambda_3 = 2$ ,  $\lambda_4 = 3$ .

(a) What is the characteristic polynomial of  $A$ ?

(b) Compute  $\text{tr}(A)$ .

(c) Compute  $\det(A)$ .

(d) What are the eigenvalues of  $A^2$ ?

(e) Compute  $\text{tr}(A^2)$ .

(f) Compute  $\det(A^2)$ .

3. 6 points Let  $D = \begin{bmatrix} -3 & 0 \\ 0 & 7 \end{bmatrix}$ .

(a) Let  $A = \begin{bmatrix} 1 & 5 \\ 5 & 3 \end{bmatrix}$ . Is  $A$  similar to  $D$ ? Explain why, or why not.

(b) Let  $B = \begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix}$ . Is  $B$  similar to  $D$ ? Explain why, or why not.

(c) Let  $C = \begin{bmatrix} -4 & -3 \\ 5 & 9 \end{bmatrix}$ . Is  $C$  similar to  $D$ ? Explain why, or why not.

4. 6 points A  $2 \times 2$  matrix  $A$  matrix has eigenvalues  $\lambda_1 = 2$  and  $\lambda_2 = 5$ , with corresponding eigenvectors  $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  and  $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ .

(a) Find  $A$ .

- (b) Consider the discrete dynamical system  $\vec{x}(t+1) = A\vec{x}(t)$ , with initial value  $\vec{x}(0) = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ .

Find a closed form for  $\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ .