Prof. Alexandru Suciu

MTH U371 **LINEAR ALGEBRA** **Spring 2006**

QUIZ 4

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1. $\boxed{\text{6 points}}$ Let $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 3 & -4 \\ -1 & 5 & -7 \end{bmatrix}$.

Determine whether the column vectors of A are dependent or independent. If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

2. $\boxed{\text{6 points}}$ For which value(s) of the constant k do the vectors below for a basis of $\mathbb{R}^4$?

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ k \end{bmatrix}, \qquad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 4 \end{bmatrix}, \qquad \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3 \end{bmatrix}, \qquad \vec{v}_4 = \begin{bmatrix} 3 \\ -2 \\ 1 \\ k \end{bmatrix}.$$

3. $\boxed{\text{8 points}}$ The matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \end{bmatrix}$ has the matrix $E = \begin{bmatrix} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ as its row-reduced echelon form.

(a) Find a basis for the image of A.

(b) Find a basis for the kernel of A.

(c) Compute $\dim(\operatorname{im} A)$ and $\dim(\ker A)$.

4. $\boxed{\text{5 points}}$ Consider the 5×4 matrix $A = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{v}_4 \end{bmatrix}$. We are told the vector $\begin{bmatrix} -5 \\ 4 \\ -3 \\ 2 \end{bmatrix}$ is in the kernel of A. Write $\vec{v}_4$ as a linear combination of $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$.