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MTH U371 **LINEAR ALGEBRA** **Spring 2005**

QUIZ 1

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1. 7 points Use Gauss-Jordan elimination to solve the following system of equations. bf Indicate for each step which row operation you use. Carry out the elimination **all the way** (to rref form) before solving.

$$\begin{array}{rcrcrcr} x & & & - & 3z & = & 8 \\ 2x & + & 2y & + & 9z & = & 7 \\ & & y & + & 5z & = & -2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -3 & \vdots & 8 \\ 2 & 2 & 9 & \vdots & 7 \\ 0 & 1 & 5 & \vdots & -2 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & \vdots & 5 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix}$$

Unique solution:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}.$$

2. 8 points Consider the following system of linear equations:

$$\begin{aligned} x_1 \quad + 2x_3 + 4x_4 &= -9 \\ x_2 - 3x_3 - x_4 &= 5 \\ -x_2 + 3x_3 + 4x_4 &= -7 \end{aligned}$$

Identify which variables are leading and which are free. Write down the general solution of the system, in **vector form**.

$$\begin{bmatrix} 1 & 0 & 2 & 4 & \vdots & -9 \\ 0 & 1 & -3 & -1 & \vdots & 5 \\ 0 & -1 & 3 & 4 & \vdots & -7 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 2 & 0 & \vdots & -\frac{19}{3} \\ 0 & 1 & -3 & 0 & \vdots & \frac{13}{3} \\ 0 & 0 & 0 & 1 & \vdots & -\frac{2}{3} \end{bmatrix}$$

Leading variables: x_1, x_2, x_4.
Free variable: x_3.
Set $x_3 = t$, and solve for the leading variables.
There are infinitely many solutions, depending on a single parameter t:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -19 \\ 13 \\ 0 \\ -2 \end{bmatrix} + t \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

3. 6 points The reduced row echelon forms of the augmented matrices of 3 systems are given below. In each case, indicate the rank of the matrix of coefficients (to the left of the dotted lines), and the number of solutions of the system (you need **not** write down the solutions.)

(a) $\left[\begin{array}{cccc:c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right]$

rank = 4
\# solutions = 1

(b) $\left[\begin{array}{cccc:c} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$

rank = 2
\# solutions = 0

(c) $\left[\begin{array}{cccc:c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 3 & 0 & 4 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

rank = 3
\# solutions = ∞

4. 3 points Find the matrix A of the linear transformation $T\colon \mathbb{R}^5 \to \mathbb{R}^3$ given by

$$y_1 = 5x_1 + 4x_2 + 3x_3 + x_5$$
$$y_2 = -8x_2 + 2x_3 - x_5$$
$$y_3 = 7x_1 - 6x_4$$

$$A = \begin{bmatrix} 5 & 4 & 3 & 9 & 1 \\ 0 & -8 & 2 & 0 & -1 \\ 7 & 0 & 0 & -6 & 0 \end{bmatrix}$$

5. 6 points Consider the linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^2$, where

$$T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}4\\5\end{bmatrix}, \qquad T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\-2\end{bmatrix}, \qquad T\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-3\\1\end{bmatrix}.$$

(a) Find the matrix A of T.

$$A = \begin{bmatrix} 4 & 0 & -3 \\ 5 & -2 & 1 \end{bmatrix}$$

(b) Compute $T\begin{bmatrix}2\\-6\\3\end{bmatrix}$.

$$T\begin{bmatrix}2\\-6\\3\end{bmatrix} = \begin{bmatrix} 4 & 0 & -3 \\ 5 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix}2\\-6\\3\end{bmatrix} = \begin{bmatrix}-1\\25\end{bmatrix}$$