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MTH U371 **LINEAR ALGEBRA** **Spring 2005**

Sample Questions for Quiz 3

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1. Let $A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 2 & 2 & 0 \\ -6 & -3 & 0 & 1 \\ 2 & 1 & -2 & 0 \end{bmatrix}$.

(a) Find a basis for $\operatorname{im} A$.

(b) Find a basis for $\ker A$.

(c) Find $\operatorname{rank} A$.

2. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ -1 & 0 & 1 & 2 & 3 \\ 2 & 3 & 0 & 5 & 8 \end{bmatrix}$.

(a) Find a basis for the image of A.

(b) Find a basis for the kernel of A.

(c) Find the rank and the nullity of A.

3. Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 4 & 5 & 2 \\ -1 & 3 & 8 \end{bmatrix}$.

(a) Determine whether the column vectors of A are dependent or independent. If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

(b) Find $\ker A$ and $\operatorname{im} A$.

(c) Does the equation $A \cdot \vec{x} = \vec{b}$ have a solution for every choice of $\vec{b}$ in $\mathbb{R}^3$? Explain your answer.

4. Are the following vectors independent or dependent? If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 6 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} -5 \\ 7 \\ -3 \end{bmatrix}$$

5. Let $A = \begin{bmatrix} 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$.

(a) Find a basis for $\operatorname{im} A$.

(b) Find a basis for $\ker A$.

(c) Compute: $\dim(\operatorname{im} A)$, $\dim(\ker A)$, $\operatorname{rank} A$.

6. Consider the folowing four vectors in $\mathbb{R}^4$.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \qquad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \qquad \vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 3 \end{bmatrix}, \qquad \vec{v}_4 = \begin{bmatrix} 2 \\ 1 \\ 7 \\ 4 \end{bmatrix}.$$

Also let A be the 4×4 matrix with columns $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$.

(a) Are the vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ independent or dependent? If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

(b) Do the vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ form a basis for $\mathbb{R}^4$? Explain your answer.

(c) Does the equation $A \cdot \vec{x} = \vec{0}$ only have the solution $\vec{x} = \vec{0}$, or does it have other solutions? Explain your answer.

(d) Does the equation $A \cdot \vec{x} = \vec{b}$ have a solution for every choice of $\vec{b}$ in $\mathbb{R}^4$? Explain your answer.

7. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & -5 & -1 \\ -1 & 4 & 5 \end{bmatrix}$.

(a) Determine whether the column vectors of A are dependent or independent. If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

(b) Does the equation $A \cdot \vec{x} = \vec{0}$ only have the solution $\vec{x} = \vec{0}$, or does it have other solutions? Explain your answer.

(c) Does the equation $A \cdot \vec{x} = \vec{b}$ have a solution for every choice of $\vec{b}$ in $\mathbb{R}^3$? Explain your answer.

8. Consider the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

(a) Are the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ linearly independent or dependent? If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

(b) Write the vector $\vec{b} = \begin{bmatrix} 1 \\ -7 \\ 5 \end{bmatrix}$ as a linear combination of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

9. Find a basis of the subspace of $\mathbb{R}^4$ defined by the equation $x_1 + 3x_2 - 5x_3 + 2x_4 = 0$.

10. [10 points] Let V be the subspace of $\mathbb{R}^3$ defined by the equation $x_1 + 2x_2 - 5x_3 = 0$.

(a) Find a basis for V.

(b) Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $\ker T = \{\vec{0}\}$ and $\operatorname{im} T = V$. Describe T by its matrix A.

11. Let V be the subspace of $\mathbb{R}^3$ defined by the equation $2x_1 - 7x_2 + x_3 = 0$. Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $\ker T = \{\vec{0}\}$ and $\operatorname{im} T = V$. Describe T by its matrix A.

12. In each of the following, a subset S of $\mathbb{R}^3$ is given. Circle one answer:

(a) $S = \{(t, 2t, 3t) \mid t \text{ is a real number}\}$

S is closed under addition:	YES	NO	MAYBE
S is closed under scalar multiplication:	YES	NO	MAYBE
S is a vector subspace of V:	YES	NO	MAYBE

(b) $S = \{(t, 2t, 3t) \mid t \text{ is a positive real number}\}$

S is closed under addition:	YES	NO	MAYBE
S is closed under scalar multiplication:	YES	NO	MAYBE
S is a vector subspace of V:	YES	NO	MAYBE

(c) $S = \{(t, 2t, 3t) \mid t \text{ is an integer}\}$

S is closed under addition:	YES	NO	MAYBE
S is closed under scalar multiplication:	YES	NO	MAYBE
S is a vector subspace of V:	YES	NO	MAYBE

(d) $S = \{(t+1, 2t, 3t-1) \mid t \text{ is a real number}\}$

S is closed under addition:	YES	NO	MAYBE
S is closed under scalar multiplication:	YES	NO	MAYBE
S is a vector subspace of V:	YES	NO	MAYBE

13. In each of the following, a subset V of $\mathbb{R}^2$ is given. Circle one answer:

(a) $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; x - 2y = 6 \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^2$: YES NO

(b) $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; \begin{matrix} x - 2y = 0 \\ x, y \text{ integers} \end{matrix} \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^2$: YES NO

(c) $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; xy \geq 0 \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^2$: YES NO

(d) $V = \left\{ \begin{bmatrix} 2x - y \\ x + 3y \end{bmatrix} \;\middle|\; \begin{matrix} x, y \text{ arbitrary} \\ \text{constants} \end{matrix} \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^2$: YES NO

14. In each of the following, a subset V of $\mathbb{R}^3$ is given. Circle one answer:

(a) $V = \left\{ \begin{bmatrix} x + y + z \\ x + z \\ y \end{bmatrix} \;\middle|\; x, y, z \text{ arbitrary constants} \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^3$: YES NO

(b) $V = \left\{ \begin{bmatrix} x + y + z \\ x + z \\ y + 1 \end{bmatrix} \;\middle|\; x, y, z \text{ arbitrary constants} \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^3$: YES NO

(c) $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \;\middle|\; x, y, z \text{ positive integers} \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^3$: YES NO

(d) $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \;\middle|\; xy \leq 0 \right\}$

Is closed under addition: YES NO
Is closed under scalar multiplication: YES NO
Is a vector subspace of $\mathbb{R}^3$: YES NO