
QUIZ 5

1. 8 points Let A, B, C be three 4×4 matrices, with $\det A = 2$, $\det B = -1$, $\det C = 0$.

(a) In the following, circle the correct answer. Provide a (short) explanation in each case.

• Is A invertible? Yes No Maybe

• Is B invertible? Yes No Maybe

• Is C invertible? Yes No Maybe

• Is A orthogonal? Yes No Maybe

• Is B orthogonal? Yes No Maybe

• Is C orthogonal? Yes No Maybe

(b) Compute $\det(A \cdot B \cdot A^T)$.

(c) Compute $\det(3A^2)$.

2. 8 points Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

(a) Find the eigenvalues of A .

(b) Find a basis for each eigenspace of A .

(c) Find a diagonal matrix D and an invertible matrix S such that $A = S \cdot D \cdot S^{-1}$. (You need not compute S^{-1} .)

3. 8 points A 2×2 matrix A has eigenvalues $\lambda_1 = 3$, $\lambda_2 = -4$.

(a) What is the characteristic polynomial of A ?

(b) Compute $\text{tr}(A)$.

(c) Compute $\det(A)$.

(d) Compute $\det(4I_2 + A)$.

(e) Is A diagonalizable? If yes, compute its diagonalization D . If not, explain why not.

(f) Let $B = \begin{bmatrix} 1 & 3 \\ 3 & -2 \end{bmatrix}$. Is B similar to A ? Explain why, or why not.

(g) Let $C = \begin{bmatrix} 5 & 6 \\ -3 & -6 \end{bmatrix}$. Is C similar to A ? Explain why, or why not.

(h) Let $M = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$. Is M similar to A ? Explain why, or why not.

4. 6 points A 2×2 matrix A matrix has eigenvalues $\lambda_1 = 6$ and $\lambda_2 = 7$, with corresponding eigenvectors $\vec{v}_1 = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$. Find A .