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**QUIZ 3**

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1. 12 points      Let  $A = \begin{bmatrix} 1 & 0 & 2 & 0 & 3 \\ 2 & 0 & 4 & -1 & 7 \\ -1 & 3 & 0 & 6 & 2 \end{bmatrix}$ .

(a) Find the row reduced echelon form of  $A$ .

(b) Find a basis for the image of  $A$ .

(c) Find a basis for the kernel of  $A$ .

(d) Find the rank and the nullity of  $A$ .

2. 10 points Consider the following four vectors in  $\mathbb{R}^4$ .

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \\ 2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 4 \\ 0 \\ -4 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \\ 4 \end{bmatrix}, \quad \vec{v}_4 = \begin{bmatrix} 0 \\ 1 \\ -5 \\ 4 \end{bmatrix}.$$

- (a) Are the vectors  $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$  independent or dependent? If they are independent, say why. If they are dependent, exhibit a linear dependence relation among them.

- (b) Do the vectors  $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$  form a basis for  $\mathbb{R}^4$ ? Explain your answer.

- (c) Do the vectors  $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$  span  $\mathbb{R}^4$ ? Explain your answer.

3. 8 points Let  $V$  be the subspace of  $\mathbb{R}^3$  defined by the equation  $2x_1 - 3x_2 + 4x_3 = 0$ .

- (a) Express  $V$  as the kernel of a matrix  $A$ .

- (b) Express  $V$  as the image of a matrix  $B$ .

- (c) Find a basis for  $V$ .