MTH U345 **Ordinary Differential Equations** **Fall 2008**

Practice Quiz 5

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1. Find the Laplace transforms $F(s)$ of the following functions $f(t)$:

(a) $f(t) = \begin{cases} 0, & t < 2 \\ (t-2)^2, & t \geq 2 \end{cases}$

(b) $f(t) = \begin{cases} 0, & t < 1 \\ t^2 - 2t + 2, & t \geq 1 \end{cases}$

(c) $f(t) = u_2(t)e^{3t-6}$

(d) $f(t) = t - u_1(t)(t-1)$

(e) $f(t) = (t-3)u_2(t) - (t-2)u_3(t)$

(f) $f(t) = e^{5t}u_2(t)$

(g) $f(t) = e^{3t}\delta_2(t) - e^{2t}\delta_3(t)$

2. Find the inverse Laplace transform $f(t)$ of the following functions $F(s)$:

(a) $F(s) = \dfrac{1}{s^2+8}$

(b) $F(s) = \dfrac{1}{s^2-10}$

(c) $F(s) = \dfrac{1-2s}{s^2+4s+5}$

(d) $F(s) = \dfrac{2s-3}{s^2+2s+10}$

(e) $F(s) = \dfrac{8s^2-4s+12}{s(s^2+4)}$

(f) $F(s) = \dfrac{e^{-2s}}{s^2+s-2}$

(g) $F(s) = \dfrac{2(s-1)e^{-2s}}{s^2-2s+2}$

(h) $F(s) = 1 + \dfrac{e^{-s}+e^{-2s}-e^{-3s}-e^{-4s}}{s}$

3. For the initial value problem $y'' + y = \cos(3t), \quad y(0) = 1,\ y'(0) = 0.$

(a) Determine the Laplace transform $Y(s)$ of the solution $y(t)$. (**Do not solve the IVP**).

(b) Find the partial fraction decomposition of $\dfrac{1}{s^3(s+1)^2}$.

4. For the initial value problem $y'' + 3y' + 2y = t, \quad y(0) = 0,\ y'(0) = 2.$

(a) Determine the Laplace transform $Y(s)$ of the solution $y(t)$. (**Do not solve the IVP**).

(b) Find the partial fraction decomposition of $\dfrac{1}{s^3(s-1)^2}$.

5. Use Laplace transforms to find the solution of the differential equation $y''+y = \sin(2t)$ satisfying the initial conditions $y(0) = 2,\ y'(0) = 1$.

6. Use Laplace transforms to solve the IVP: $y'' - 2y' + 2y = e^{-t},\ y(0) = 0,\ y'(0) = 1.$

7. Use Laplace transforms to solve the IVP: $y'' + 2y' + y = 4e^{-t},\ y(0) = 2,\ y'(0) = -1.$

8. Solve the IVP: $y'' + 3y' + 2y = u_2(t),\ y(0) = 0,\ y'(0) = 1.$

9. Solve the IVP: $y'' + 4y = 15e^{t-2}u_2(t),\ y(0) = 0,\ y'(0) = 0.$

10. Solve the IVP: $2y'' + y' + 2y = \delta_5(t),\ y(0) = 0,\ y'(0) = 0.$

11. Solve the IVP: $y'' + 2y' + 2y = \delta_\pi(t),\ y(0) = 1,\ y'(0) = 0.$