MTH U345 **Ordinary Differential Equations** **Fall 2008**

Solutions to Practice Quiz 5

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2. Find the inverse Laplace transform $f(t)$ of the following functions $F(s)$:

(c) $F(s) = \dfrac{1-2s}{s^2+4s+5} \quad \longrightarrow \quad f(t) = e^{-2t}(-2\cos(2t) + 5\sin(2t))$

(d) $F(s) = \dfrac{2s-3}{s^2+2s+10} \quad \longrightarrow \quad f(t) = e^{-t}(2\cos(3t) - \frac{5}{3}\sin(3t))$

(e) $F(s) = \dfrac{8s^2-4s+12}{s(s^2+4)} \quad \longrightarrow \quad f(t) = 3 - 2\sin(2t) + 5\cos(2t))$

(f) $F(s) = \dfrac{e^{-2s}}{s^2+s-2} \quad \longrightarrow \quad f(t) = \frac{1}{3}u_2(t)(e^{t-2} - e^{-2(t-2)}$

(g) $F(s) = \dfrac{2(s-1)e^{-2s}}{s^2-2s+2} \quad \longrightarrow \quad f(t) = 2u_2(t)e^{t-2}\cos(t-2)$

(h) $F(s) = 1 + \dfrac{e^{-s}+e^{-2s}-e^{-3s}-e^{-4s}}{s} \quad \longrightarrow \quad f(t) = \delta_0(t) + u_1(t) + u_2(t) - u_3(t) + u_4(t)$

4. For the initial value problem $y'' + 3y' + 2y = t, \quad y(0) = 0,\ y'(0) = 2.$

(a) Determine the Laplace transform $Y(s)$ of the solution $y(t)$.

$$Y(s) = \frac{1+2s^2}{s^2(s+1)(s+2)}$$

(b) Find the partial fraction decomposition of $\dfrac{1}{s^3(s-1)^2}$.

$$\frac{1}{s^3(s-1)^2} = \frac{1}{s^3} - \frac{2}{s^2} + \frac{3}{s} + \frac{1}{(s-1)^2} - \frac{3}{s-1}$$

5. Use Laplace transforms to find the solution of the differential equation $y''+y = \sin(2t)$ satisfying the initial conditions $y(0) = 2,\ y'(0) = 1$.

$$y(t) = 2\cos(t) + \frac{5}{3}\sin(t) - \frac{1}{3}\sin(2t)$$

6. Use Laplace transforms to solve the IVP: $y'' - 2y' + 2y = e^{-t},\ y(0) = 0,\ y'(0) = 1$.

$$y(t) = \frac{1}{5}\left(e^{-t} - e^t\cos t + 7e^t\sin t\right)$$

7. Use Laplace transforms to solve the IVP: $y'' + 2y' + y = 4e^{-t},\ y(0) = 2,\ y'(0) = -1$.

$$y(t) = e^{-t}(2 + t + 2t^2)$$

9. Solve the IVP: $y'' + 4y = 15e^{t-2}u_2(t),\ y(0) = 0,\ y'(0) = 0$.

$$y(t) = 3u_2(t)\left(e^{t-2} - \cos(4-2t) + \tfrac{1}{2}\sin(4-2t)\right)$$

10. Solve the IVP: $2y'' + y' + 2y = \delta_5(t),\ y(0) = 0,\ y'(0) = 0$.

$$y(t) = \frac{2}{\sqrt{15}}e^{-(t-5)/4}\sin\left(\frac{\sqrt{15}}{4}(t-5)\right)$$