MTH U345 | **Ordinary Differential Equations** | **Fall 2008**

Practice Quiz 4

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1. Find the general solution of the differential equation $y'' - 7y' + 12y = 5e^{3t}$.

2. Solve the initial value problem $y'' - 3y' + 2y = t,\ y(0) = 0,\ y'(0) = 0$.

3. Solve the initial value problem $y'' + 4y' + 13y = 0,\ y(0) = 5,\ y'(0) = 2$.

4. (a) Find the general real-valued solution of the differential equation $y'' + 6y' + 25y = 0$.

(b) Find the solution of the differential equation $y'' + 6y' + 25y = 2e^{-5t}$ such that $y(0) = 1/2$ and $y'(0) = 0$. (Make use of your answer to part (a).)

5. Solve the initial value problem $y'' + 9y = \cos(2t),\ y(\pi) = 1,\ y'(\pi) = 2$.

6. Solve the initial value problem $y'' + 9y = \cos(3t),\ y(0) = -1,\ y'(0) = 6$.

7. Consider the differential equation $y'' + 9y = \cos(3.2t)$.

(a) Determine the frequency of the beats.

(b) Determine the frequency of the rapid oscillations.

(c) Determine the maximum amplitude of the oscillations.

(d) Use the information from parts (a), (b), (c) to give a rough sketch of the typical solution. (Indicate the periods and the amplitude on the graph.)

8. Consider the system $\dfrac{dx}{dt} = x(1 - x - y),\ \dfrac{dy}{dt} = y(3 - 2x - y)$.

(a) Find the equilibrium points.

(b) Find the Jacobian matrix of the system.

(c) Find the linearized system for each of the equilibrium points from part (a).

(d) Sketch the phase portraits of the linearized systems from part (d).

(e) Classify each equilibrium point as either source, sink, saddle point, center, etc.

9. Consider the system $\dfrac{dx}{dt} = x(2 - x - y),\ \dfrac{dy}{dt} = y(4 - x^2 - y^2)$.

(a) Find the equilibrium points.

(b) Find the Jacobian matrix of the system.

(c) Find the linearized system for each of the equilibrium points from part (a).

(d) Sketch the phase portraits of the linearized systems from part (d).

(e) Classify each equilibrium point as either source, sink, saddle point, center, etc.