MTH U345 **Ordinary Differential Equations** **Fall 2008**

Solutions to Quiz 3

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1. $Y' = AY$, with $A = \begin{bmatrix} 5 & -2 \\ 1 & 2 \end{bmatrix}$.

(a) Eigenvalues: $\lambda_1 = 3$, $\lambda_2 = 4$.

(b) Eigenvectors: $V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $V_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(c) General solution: $Y(t) = C_1 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{4t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(d) Solution with $Y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$: $Y(t) = -3e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2e^{4t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

2. The matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ has eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$, with corresponding eigenvectors $V_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $V_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

The phase portrait consists of lines L_1 and L_2 through the vectors V_1 and V_2, with contraction on L_1 expansion on L_2, and other solution curves curving in the plane accordingly, coming from the direction of L_1 and going in the direction of L_2. The origin is a saddle point.

3. $Y' = AY$, with $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$.

(a) Eigenvalues: $\lambda_1 = 0$, $\lambda_2 = 4$.

(b) Eigenvectors: $V_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $V_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(c) General solution: $Y(t) = C_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} + C_2 e^{4t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(d) The phase portrait consists of the line L_1 through the vector V_1 (the points on this line are precisely the equilibrium points), together with rays emanating from points on L_1, in a direction parallel to V_2, and pointing away from L_1.

4. $Y' = AY$, with $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$.

(a) Unique eigenvalue: $\lambda = 2$

(b) Let $V_0 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Corresponding eigenvector: $V_1 = (A - \lambda I)V_0 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$.

(c) Solution with $Y(0) = V_0$: $Y(t) = e^{2t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + te^{2t} \begin{bmatrix} 3 \\ 3 \end{bmatrix}$.