MTH U345 **Ordinary Differential Equations** **Fall 2008**

Practice Quiz 3

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1. Consider the linear system $Y' = AY$, with $A = \begin{bmatrix} -2 & -3 \\ 1 & -6 \end{bmatrix}$.

(a) Find the eigenvalues λ_1 and λ_2 of the matrix A.

(b) Find (non-zero) eigenvectors V_1 and V_2 corresponding to each eigenvalue.

(c) Find the general solution $Y = Y(t)$ of the given system.

(d) Find the solution $Y = Y(t)$ with initial condition $Y(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

(e) Sketch the phase portrait for the system, indicating the straight-line solution(s), and several other solution curves, including the one found in part (d). What kind of equilibrium point is the origin?

2. Consider the linear system $Y' = AY$, with $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$.

(a) Find the eigenvalues λ_1 and λ_2 of the matrix A.

(b) Find (non-zero) eigenvectors V_1 and V_2 corresponding to each eigenvalue.

(c) Find the general solution $Y = Y(t)$ of the given system.

(d) Find the solution $Y = Y(t)$ with initial condition $Y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(e) Sketch the phase portrait for the system, indicating the straight-line solutions, and several other solution curves, including the one found in part (d). What kind of equilibrium point is the origin?

3. Consider the linear system $Y' = AY$, with $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$.

(a) Find the eigenvalues λ_1 and λ_2 of the matrix A.

(b) Find (non-zero) eigenvectors V_1 and V_2 corresponding to each eigenvalue.

(c) Find the general solution $Y = Y(t)$ of the given system.

(d) Find the solution $Y = Y(t)$ with initial condition $Y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(e) Sketch the phase portrait for the system, indicating the straight-line solutions, and several other solution curves, including the one found in part (d). What kind of equilibrium point is the origin?

4. Consider the linear system $Y' = AY$, with $A = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$.

(a) Find the eigenvalues λ_1 and λ_2 of the matrix A.

(b) Find (non-zero) eigenvectors V_1 and V_2 corresponding to each eigenvalue.

(c) Find the general solution $Y = Y(t)$ of the given system.

(d) Find the solution $Y = Y(t)$ with initial condition $Y(0) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$.

(e) Sketch the phase portrait for the system, indicating the straight-line solutions, and several other solution curves, including the one found in part (d). What kind of equilibrium point is the origin?

5. Consider the linear system $Y' = AY$, with $A = \begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix}$.

(a) It turns out that A has a single eigenvalue, λ. Compute this eigenvalue.

(b) Given an initial condition $V_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$, find the corresponding eigenvector V_1 for the eigenvalue λ.

(c) Find the general solution $Y = Y(t)$ of the given system.

(d) Find the solution $Y = Y(t)$ with initial condition $Y(0) = V_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(e) Sketch the phase portrait for the system, indicating the straight-line solution, and several other solution curves, including the one found in part (d). What are the equilibrium solutions?

6. Let $A = \begin{bmatrix} -1 & -2 \\ 4 & -5 \end{bmatrix}$.

(a) Find the eigenvalues of A.

(b) Based on your answer to part (a), describe in words the behavior of the solution Y to $Y' = AY$ if $Y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. (Do NOT solve the differential equation.)

(c) What kind of equilibrium point is the origin?

7. Consider the linear system $Y' = AY$, with $A = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$.

(a) Find the general solution $Y = Y(t)$.

(b) Find the solution $Y = Y(t)$ with initial condition $Y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(c) Sketch the $x(t)$- and $y(t)$-graphs of this particular solution.

(d) What kind of equilibrium point is the origin?