MTH U345 **Ordinary Differential Equations** **Fall 2008**

Practice Quiz 2: Answer Key

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1. Solve the initial value problem $4y'' - 12y' + 9y = 0,\ y(0) = 9,\ y'(0) = 8.$

$$y(t) = 9e^{3t/2} - \frac{11}{2}te^{3t/2}.$$

2. If $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is a solution of $X' = AX$ where $A = \begin{bmatrix} 6 & -4 \\ 10 & 2 \end{bmatrix}$, give x_1' and x_2' at the point where $x_1 = 5$ and $x_2 = -3$.

$$X' = \begin{bmatrix} 42 \\ 44 \end{bmatrix}$$

3. Two of the three vector-valued functions

$$U(t) = e^{-2t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad V(t) = e^{2t}\begin{bmatrix} -2 \\ 1 \end{bmatrix}, \quad W(t) = e^{3t}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

are solutions to the system $Y' = BY$ where $B = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$.

(a) Determine which two functions are solutions.

$$U' = -2U, \quad BU(t) = e^{-2t}\begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = e^{-2t}\begin{bmatrix} -2 \\ 2 \end{bmatrix} = -2U. \ \ U \text{ solves.}$$

$$V' = 2V, \quad BV(t) = e^{2t}\begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix} = e^{2t}\begin{bmatrix} 0 \\ -3 \end{bmatrix} \neq V'. \ \ V \text{ does not solve.}$$

$$W' = 3W, \quad BW(t) = e^{3t}\begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix} = e^{3t}\begin{bmatrix} 12 \\ 3 \end{bmatrix} = 3W. \ \ W \text{ solves.}$$

The solutions are: U and W.

(b) Using the two functions which are solutions, find a third solution Y satisfying the initial condition $Y(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

The solution will be that linear combination $Y = k_1U + k_2W$ of U and W which has the specified initial conditions. At $t = 0$ this says

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = Y(0) = k_1U(0) + k_2W(0) = k_1\begin{bmatrix} 1 \\ -1 \end{bmatrix} + k_2\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} k_1 + 4k_2 \\ -k_1 + k_2 \end{bmatrix}.$$

Thus $k_1 + 4k_2 = 0$ and $-k_1 + k_2 = 1$. Solving this system gives $k_1 = -4/5$ and $k_2 = 1/5$. Thus the solution is

$$Y(t) = -(4/5)U(t) + (1/5)W(t) = \frac{1}{5}\begin{bmatrix} -4e^{-2t} + 4e^{3t} \\ 4e^{-2t} + e^{3t} \end{bmatrix}$$