

Quiz 1

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1. 10 points Solve the initial value problem

$$\frac{dy}{dt} = t + ty^2, \quad y(0) = 1$$

For which values of  $t$  is the solution defined?

2. 10 points Given the differential equation  $\frac{dy}{dt} = (y^2 - 4)(y - 3)$ .

(a) Sketch the phase line.

(b) Identify the equilibrium points as sinks, sources, or nodes.

(c) For each of the following initial conditions, sketch the corresponding curve in the phase plane, making sure you indicate the initial position on the plane.

$$y(0) = -\frac{5}{2}, \quad y(1) = -1, \quad y(0) = 0, \quad y(1) = \frac{5}{2}, \quad y(0) = \frac{7}{2}.$$

3. 10 points Solve the initial value problem

$$\frac{dy}{dt} + \frac{3}{t}y = \frac{1}{t^4}, \quad y(1) = 1$$

For which values of  $t$  is the solution defined?

4. 10 points A 100 gallon tank initially contains 20 gallons of pure water. A salt water solution containing 4 pounds of salt per gallon enters the tank at 7 gallons per minute, and the mixture kept uniform by stirring, flows out at the rate of 5 gallons per minute.
- (a) How many gallons of salt water solution are there after  $t$  minutes?
  - (b) When will the tank be full?
  - (c) Write down the initial value problem that describes the quantity of salt,  $S(t)$  kg, at time  $t$ .  
**YOU DO NOT NEED TO SOLVE FOR  $S(t)$ .**