MTH U345 | **Ordinary Differential Equations** | **Fall 2008**

Practice Quiz 1

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1. Solve: $\dfrac{dy}{dt} - \dfrac{2}{t}y = t^2 \sin t.$

2. Solve: $(t+1)\dfrac{dy}{dt} = 1 + y^2.$

3. Solve: $\dfrac{dx}{dt} + \dfrac{1}{2t}x = \dfrac{1}{2}.$

4. Solve: $\dfrac{dy}{dt} - ty = ty^3.$

5. Solve: $\dfrac{dx}{dt} + \dfrac{2}{t}x = \dfrac{1}{t^3} + 3, \quad x(1) = 3.$

6. Solve: $\dfrac{dy}{dt} + \dfrac{3}{t}y = 2, \quad y(2) = 1.$

7. Solve: $\dfrac{dy}{dt} = \dfrac{e^{2t}}{2y}, \quad y(0) = -1.$

8. Given the autonomous differential equation $\dfrac{dy}{dt} = (y-1)^2(y+3)^3.$

(a) Sketch the phase line.

(b) Identify the equilibrium points as sinks, sources, or nodes.

(c) For each of the following initial conditions, sketch the corresponding curve in the phase plane.

$$y(0) = -4, \quad y(0) = 0, \quad y(1) = 0, \quad y(0) = 2.$$

9. A 300 gallon tank initially contains 100 gallons of pure water. A salt water solution containing 3 pounds of salt per gallon enters the tank at 8 gallons per minute, and the mixture is removed at the rate of 6 gallons per minute. How many pounds of salt is in the tank when the tank is full?

10. A tank initially contains 20 kg of salt dissolved in 200 liters of water. A brine solution containing 3 kg of salt/liter flows into the tank at the rate of 2 liters/minute. The mixture, kept uniform by stirring, flows out at the rate of 4 liters/minute.

(a) Write down the initial value problem that describes the quantity of salt, $S(t)$ kg, at time t minutes.

(b) Find $S(t)$.

(c) What is the quantity of salt after 30 minutes?

11. A microcosm contains a population of microorganisms whose birth and death rates are proportional to the square root of the population. Initially there is just 1 microorganism, but after 5 hours there are 4 of them.

(a) Write down the initial value problem that describes the population of microorganisms, $P(t)$, at time t hours.

(b) Find $P(t)$.

(c) When will the microorganisms take over the microcosm?