Prof. Alexandru Suciu

MATH 3175 **Group Theory** **Fall 2010**

Quiz 4

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1. (5 points) Let $\mathbb{R}$ be the additive group of real numbers, and let $\mathbb{R}^+$ be the multiplicative group of positive real numbers. Consider the map $\phi\colon \mathbb{R} \to \mathbb{R}^+$ given by $\phi(x) = 2^x$.

(a) Show that ϕ is an isomorphism from $\mathbb{R}$ to $\mathbb{R}^+$.

(b) What is the inverse isomorphism?

2. (4 points) Show that the automorphism group $\mathrm{Aut}(\mathbb{Z}_{10})$ is isomorphic to a cyclic group $\mathbb{Z}_n$. What is n?

3. (6 points) Show that the following pairs of groups are *not* isomorphic. In each case, explain why.

(a) $U(12)$ and $\mathbb{Z}_4$.

(b) S_3 and $\mathbb{Z}_6$.

(c) S_4 and D_{12}.

4. (5 points) Let G be a group, and let a be an element of order 30. How many left cosets of $\langle a^6 \rangle$ in $\langle a \rangle$ are there? List all these cosets. (Make sure to indicate all the elements in each coset.)

5. (5 points) Let S_3 be the group of permutations of the set $\{1, 2, 3\}$. Consider the subgroup $H = \{(1), (13)\}$.

(a) Write down all the left cosets of H in S_3. (Make sure to indicate all the elements in each coset.)

(b) What is the index of H in S_3?

6. (5 points) Suppose G is a group of order 35.

(a) What are the possible orders for the elements of G?

(b) Suppose G has an element of order 35. What is G?

(c) Suppose G has precisely one subgroup of order 5, and one subgroup of order 7. What is G?