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MATH 3175 | **Group Theory** | **Fall 2010**

Practice Quiz 5

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1. Prove, by comparing orders of elements, that the following pairs of groups are not isomorphic:

(a) $\mathbb{Z}_8 \oplus \mathbb{Z}_4$ and $\mathbb{Z}_{16} \oplus \mathbb{Z}_2$.

(b) $\mathbb{Z}_9 \oplus \mathbb{Z}_9$ and $\mathbb{Z}_{27} \oplus \mathbb{Z}_3$.

2. Describe a specific isomorphism $\phi\colon \mathbb{Z}_6 \oplus \mathbb{Z}_5 \to \mathbb{Z}_{30}$.

3. Describe a specific isomorphism $\psi\colon U(16) \to \mathbb{Z}_2 \oplus \mathbb{Z}_4$.

4. Prove or disprove that $D_6 \cong D_3 \oplus \mathbb{Z}_2$.

5. Prove or disprove that $D_{12} \cong D_4 \oplus \mathbb{Z}_3$.

6. How many elements of order 6 are there in $\mathbb{Z}_6 \oplus \mathbb{Z}_9$?

7. How many elements of order 25 are there in $\mathbb{Z}_5 \oplus \mathbb{Z}_{25}$?

8. How many elements of order 3 are there in $\mathbb{Z}_{300000} \oplus \mathbb{Z}_{900000}$?

9. Let p be a prime. Determine the number of elements of order p in $\mathbb{Z}_{p^2} \oplus \mathbb{Z}_{p^2}$.

10. Let $G = S_3 \oplus \mathbb{Z}_5$. What are all possible orders of elements in G? Prove that G is *not* cyclic.

11. The group $S_3 \oplus \mathbb{Z}_2$ is isomorphic to one of the following groups: $\mathbb{Z}_{12}$, $\mathbb{Z}_6 \oplus \mathbb{Z}_2$, A_4, D_6. Determine which one, by a process of elimination.

12. Describe all abelian groups of order $1{,}008 = 2^4 \cdot 3^2 \cdot 7$. Write each such group as a direct product of cyclic groups of prime power order.

13. Describe $U(1{,}008)$ as a direct product of cyclic groups.

14. Describe $U(195)$ as a direct product of cyclic groups in four different ways.

15. For each of the following groups, compute the number of elements of order 1, 2, 4, 8, and 16:

$$\mathbb{Z}_{16}, \quad \mathbb{Z}_8 \oplus \mathbb{Z}_2, \quad \mathbb{Z}_4 \oplus \mathbb{Z}_4, \quad \mathbb{Z}_4 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2.$$

16. List all abelian groups (up to isomorphism) of order 360.

17. (a) List the five partitions of 4, and the abelian groups of order 81 that correspond to them.

(b) A certain abelian group G of order 81 has no elements of order 27, and 54 elements of order 9. Which group is it? Why?

18. How many abelian groups (up to isomorphism) are there

(a) of order 21?

(b) of order 105?

(c) of order 210?

(d) of order 25?

(e) of order 125?

(f) of order 625?

19. Let G be a finite abelian group of order n.

(a) Suppose n is divisible by 10. Show that G has a cyclic subgroup of order 10.

(b) Suppose n is divisible by 9. Show, by example, that G need not have a cyclic subgroup of order 9.

20. Suppose G is an abelian group of order 168, and that G has exactly three elements of order 2. Determine the isomorphism class of G.