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MATH 3175 **Group Theory** **Fall 2010**

Practice Quiz 4

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1. Write down all the automorphisms of the group $\mathbb{Z}_5$.

2. Let $\mathbb{R}^+$ be the multiplicative group of positive real numbers. Show that the map $x \mapsto \sqrt[3]{x}$ is an automorphism of $\mathbb{R}^+$.

3. Show that the map $x \mapsto e^x$ is an isomorphism from $(\mathbb{R}, +)$ to $(\mathbb{R}^+, \cdot)$.

4. For each of the following pairs of groups, decide whether they are isomorphic or not. In each case, give a brief reason why.

(a) $U(5)$ and $U(10)$.

(b) $U(8)$ and $\mathbb{Z}_4$.

(c) $U(10)$ and $\mathbb{Z}_4$.

(d) S_3 and $\mathbb{Z}_6$.

(e) S_3 and D_3.

(f) A_4 and D_6.

5. Let $\phi\colon G \to H$ be an isomorphism between two groups. Suppose G is abelian. Show that H is also abelian.

6. Let g and h be two elements in a group G, and let ϕ_g and ϕ_h be the corresponding inner automorphisms. Suppose $\phi_g = \phi_h$. Show that $h^{-1}g$ belongs to the center of G.

7. Let G be a finite group, H a subgroup of G, and K a subgroup of H. Show that $|G : K| = |G : H| \cdot |H : K|$.

8. Let G be a group, and let a be an element of order 24. How many left cosets of $\langle a^{10} \rangle$ in $\langle a \rangle$ are there? List all these cosets.

9. Let D_4 be dihedral group of order 8 (the group of symmetries of the square), let $H = \langle R_1 \rangle$ be the subgroup generated by a counter-clockwise rotation by $90°$, and let $K = \langle S_0 \rangle$ be the subgroup generated by a reflection across the horizontal axis.

(a) Write down all the left cosets of H in D_4.

(b) Write down all the right cosets of H in D_4.

(c) Write down all the left cosets of K in D_4.

(d) Write down all the right cosets of K in D_4.

(e) Compute the indices $|D_4 : H|$ and $|D_4 : K|$.

10. Let S_4 be the group of permutations of the set $\{1, 2, 3, 4\}$, and let A_4 the subgroup of even permutations.

(a) Write down all the left cosets of A_4 in S_4.

(b) Write down all the right cosets of A_4 in S_4.

(c) What is the index of A_4 in S_4?

11. Suppose a group contains elements of orders 1 through 9. What is the minimum possible order of the group?

12. Suppose K is a subgroup of H, and H is a subgroup of G. If $|K| = 30$ and $|G| = 300$, what are the possible values for $|H|$?

13. Suppose $|G| = 21$, and G has precisely one subgroup of order 3, and one subgroup of order 7. Show that G is cyclic.

14. Let $G = \{(1), (13), (24), (12)(34), (13)(24), (14)(23), (1234), (1432)\}$. For each integer i from 1 to 4, find the stabilizer of i and the orbit of i.