Prof. Alexandru Suciu

MATH 3175 | **Group Theory** | **Fall 2010**

Practice Quiz 3

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1. (a) Find the subgroup lattice of $\mathbb{Z}_{36}$.

(b) Make a table with all the elements of $\mathbb{Z}_{36}$, grouped according to their orders.

(c) What are all the possible orders, and how many elements of each possible order are there?

2. (a) List of the elements of $\mathbb{Z}_{40}$ that have order 10.

(b) Suppose $|x| = 10$. List of the elements of $\langle x \rangle$ that have order 10.

3. Let G be a group, and H a subgroup of G. For any fixed $x \in G$, define the *conjugate* of H by x to be

$$xHx^{-1} = \{xhx^{-1} \mid h \in H\}.$$

Show that xHx^{-1} is a subgroup of G.

4. Let G be a group, and H a subgroup of G. Define the *normalizer* of H to be

$$N(H) = \{x \in G \mid xHx^{-1} = H\}.$$

Show that $N(H)$ is a subgroup of G.

5. Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$.

(a) Find $\alpha\beta$ and $\beta\alpha$.

(b) Compute the inverses of α, β, $\alpha\beta$, and $\beta\alpha$.

(c) Write α, β, $\alpha\beta$, and $\beta\alpha$ as products of disjoint cycles.

(d) Write α, β, $\alpha\beta$, and $\beta\alpha$ as products of transpositions.

(e) Find the orders of α, β, $\alpha\beta$, and $\beta\alpha$.

(f) Find the parity of α, β, $\alpha\beta$, and $\beta\alpha$.

6. Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$.

(a) Find $\alpha\beta$ and $\beta\alpha$.

(b) Compute the inverses of α, β, $\alpha\beta$, and $\beta\alpha$.

(c) Write α, β, $\alpha\beta$, and $\beta\alpha$ as products of disjoint cycles.

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(e) Find the orders of α, β, $\alpha\beta$, and $\beta\alpha$.

(f) Find the parity of α, β, $\alpha\beta$, and $\beta\alpha$.

7. (a) Find the conjugate of $(1234)(56)$ by $a = (25)$ in S_7.

(b) Find the conjugate of $(1234)(56)$ by $a = (27)$ in S_7.

(c) Find the conjugate of $(1234)(56)$ by $a = (37)$ in S_7.

8. How many permutations of order 5 are there in S_7?

9. How many permutations of order 6 are there in S_{10}?

10. Let α and β be two permutations in S_n.

(a) Show that $\alpha\beta\alpha^{-1}\beta^{-1}$ is an even permutation.

(b) Show that $\alpha\beta$ is even if and only if α and β are both even, or both odd.

11. Let $\beta \in S_7$, and suppose $\beta^4 = (2143567)$. Find β.

12. Find permutations α and β such that:

(a) $|\alpha| = 2$, $|\beta| = 2$, and $|\alpha\beta| = 3$.

(b) $|\alpha| = 3$, $|\beta| = 3$, and $|\alpha\beta| = 5$.