Prof. Alexandru Suciu

MTH G321 **TOPOLOGY 3** **Spring 2009**

HOMEWORK 2

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1. Let $p\colon E \to B$ be a fibration.

(a) Show that $p(E)$ is a union of path components of B.

(b) Suppose that B is path-connected. Show that any two fibers of p have the same homotopy type.

(c) Suppose that B is path-connected, and some fiber of p is path-connected. Show that E is also path-connected.

2. Let $p\colon E \to B$ be a fibration. Suppose $f\colon E \to X$ and $g\colon X \to B$ are maps such that $p = g \circ f$. For parts (a) and (b) below, prove that the stated implication holds, or give a counterexample.

(a) If g is injective, then f is a fibration.

(b) If f is surjective, then g is a fibration.

3. Let $p\colon E \to B$ be a fibration, with path-connected base space B. Choose basepoints $b_0 \in B$ and $e_0 \in F := p^{-1}(b_0)$, and let $i\colon F \to E$ be the inclusion map. Show that, for any pointed space (X, x_0), the sequence of pointed sets

$$[(X, x_0), (F, e_0)] \xrightarrow{i_\sharp} [(X, x_0), (E, e_0)] \xrightarrow{p_\sharp} [(X, x_0), (B, b_0)]$$

is exact.

4. Let ΣZ denote the unreduced suspension of a space Z, and let $X * Y$ denote the join of spaces X and Y.

(a) Given a continuous map $\phi\colon X \times Y \to Z$, show that the function $X \times Y \times I \to Z \times I$, $(x, y, t) \mapsto (\phi(x, y), t)$ induces a continuous map $h_\phi\colon X * Y \to \Sigma Z$.

(b) Suppose $\phi\colon X \times X \to X$ is a continuous map such that the maps $L_x\colon X \to X$, $L_x(y) = xy$ and $R_x\colon X \to X$, $R_x(y) = yx$ are homeomorphisms, for all $x \in X$. Show that the map $h_\phi\colon X * X \to \Sigma X$ is a bundle projection, with fiber X.

(c) Use Part (b) to conclude that the Hopf maps $S^3 \to S^2$ and $S^7 \to S^4$ are bundle projections, with fiber S^1 and S^3, respectively.