**Prof. Alexandru Suciu**

**MTH G321** **TOPOLOGY 3** **Spring 2009**

# HOMEWORK 1

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**1.** Let $f\colon A \to X$ be a cofibration. Prove:

(a) $f$ is injective.

(b) The co-restriction $f\colon A \to f(A)$ is a homeomorphism.

(c) If $X$ is Hausdorff, then $f(A)$ is a closed subset of $X$.

**2.** Let $f\colon A \to X$ be a cofibration. Suppose $A$ is contractible. Show that the quotient map, $q\colon X \to X/A$ is a homotopy equivalence.

**3.** Let $f\colon A \to X$ be a cofibration, where $A$ and $X$ are locally compact Hausdorff spaces, and let $Y$ be an arbitrary space. Then the map $p\colon Y^X \to Y^A$ defined by $p(g) = g \circ f$ is a fibration.

**4.** Let $Y$ be a space, and let $p\colon Y^I \to Y \times Y$ be the map $p(\omega) = (\omega(0), \omega(1))$, for $\omega\colon I \to Y$. Show that $p$ is a fibration.

**5.** Let $X$ be a well-pointed space. Show that the path-space $PX$ and the loop-space $\Omega X$ are both well-pointed spaces.