Torelli problem for arrangements of hypersurfaces

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Special session
Geometry and Topology of Arrangements of Hypersurfaces

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Riassunto

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4. Torelli theorem

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  Notation. $S = \mathbb{C}[x_0, \dots, x_n]$. f_i equation of D_i. $f = \prod f_i$. $d_i = \deg(f_i)$. $d = \sum d_i$. $\check{D}_i = [f_i] \in \mathbb{P}(S_i)$.

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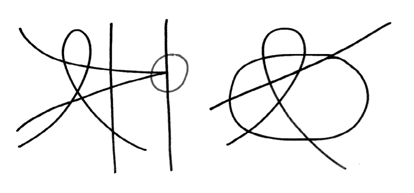
- D can have normal crossings (NC) or not.



Logarithmic vector fields

Bundle (or sheaf) $\mathcal{T}(-\log D)$.

- $\mathcal{T}(-\log D) =$ *vector fields with logarithmic poles along* D.
  Sheaf associated with S*-module of logarithmic derivations of* D:

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  $\mathcal{T}(-\log D)$ sheafification of $\mathrm{Der}(D)/\sum x_i \partial_i$.
- Syzygy of Jacobian ideal J_D.

$$0 \to \mathcal{T}(-\log D) \to \mathcal{O}_{\mathbb{P}^n}^{n+1} \xrightarrow{(\partial_0 f, \dots, \partial_n f)} J_D(d-1) \to 0.$$

Logarithmic 1-forms

Meromorphic 1-forms with logarithmic poles along D is:

$$\Omega(\log D) = \mathcal{T}(-\log D)^*(-1).$$

For NC arrangements: residue exact sequence

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$\nu : \tilde{\mathbb{P}}^n \to \mathbb{P}^n$ log-resolution of D. Smooth $\tilde{D} \to D$.

$$0 \to \Omega \to \tilde{\Omega}(\log D) \to \nu_* \mathcal{O}_{\tilde{D}} \to 0.$$

Resolution of $\mathcal{T}(-\log D)$

Theorem (Ancona)

If D is NC, then we have a (perhaps non-minimal) resolution:

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- In terms of $\Omega(\log D)$:
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- Moduli space of semistable bundle with fixed Chern polynomial c is a projective variety $M(c)$. $\check{D}_i \in \mathbb{P}(S_i)$. Rational map:

$$\omega : \prod \mathbb{P}(S_i) \dashrightarrow M(c). \qquad (\check{D}) \mapsto \Omega(\log D).$$

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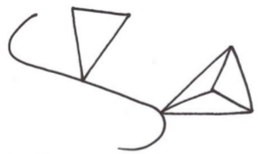
3. hyperplane arrangement is Torelli iff $\check{D}_i$ not on a KW variety: minors of $\mathcal{O}^n(-1) \to \mathcal{O}^2$ (D.F.-Matei-Vallès).



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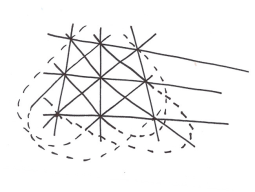
Torelli fails for $\Omega(\log D)$ more often than $\tilde{\Omega}(\log D)$.

Free divisors

9 flexes of a smooth plane cubic.



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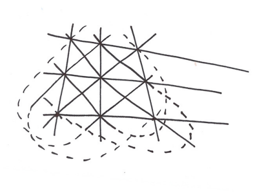

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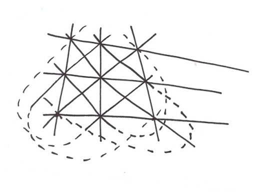

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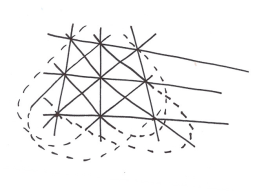

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- Difference of 12 between $c_2(\Omega(\log D))$ and $c_2(\tilde{\Omega}(\log D))$.

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- Totally non-Torelli non-NC arrangements iff $\check{D}$ contained in a tree of rational curves.

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5. Normal bundle sequence:
$$0 \to \Omega_{\mathbb{P}^n}(\log D)^* \to \Omega_{\mathbb{P}^N}(\log \mathcal{D})^*|_{V_d^n} \to \mathcal{N}_{V_d^n} \to 0$$

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8. Tensor product of $\mathcal{O}_H \otimes \mathcal{I}_{V_d^n}$ and of *Steiner resolution*:

$$0 \to \Omega_{\mathbb{P}^n}(\log D)^* \to \mathcal{O}_{\mathbb{P}^N}^{m-1} \to \mathcal{O}_{\mathbb{P}^N}^{m-N-1}(1) \to 0.$$

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10. *Reduce* by divisor $D' \subset D$ of highest degree d and iterate.

$$D' = \bigcup_{\deg(D_i)=d} D_i,$$

$$0 \to \Omega(\log(D \setminus D')) \to \Omega(\log D) \to \bigoplus_{\deg(D_i)=d} \mathcal{O}_{D_i} \to 0.$$

Logarithmic derivations for two conics

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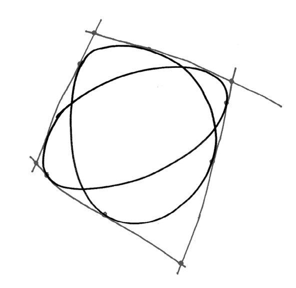

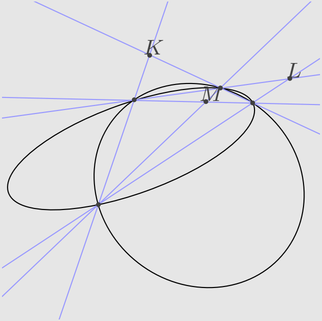

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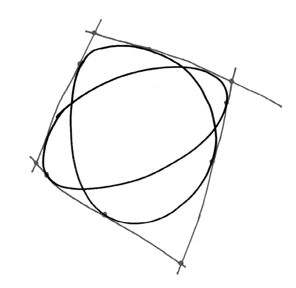

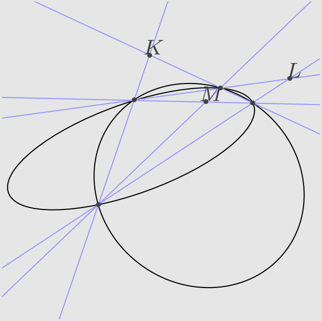

Theorem (Angelini)

D' gives $\mathcal{T}(-\log D') \simeq \mathcal{T}(-\log D)$ iff 4 bitangents to D' are $H_1, \ldots, H_4$.

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- Study arrangements having J_D of low projective dimension (e.g. free arrangements $\mathrm{pd} = 1$ and so on).
- What is $\Omega(\log D)$ when D is an invariant hypersurface? Example: discriminant of binary forms, determinant of $n \times n$ matrices, etc.