Medieval Eclipse Prediction: A Parallel Bias in Indian and Chinese Astronomy

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ABSTRACT

Since lunar and solar parallax play a crucial role in predicting solar eclipses, the focus of this paper is on the computation of parallax. A brief history of parallax computation in India and China is traced. Predictions of solar eclipses based on Nilakantha’s Tantrasangraha are statistically analyzed. They turn out to be remarkably accurate, but there is a pronounced bias towards predicting false positives rather than false negatives. The false positives occur more to the south of the ecliptic at northerly terrestrial latitudes and more to the north of the ecliptic at southerly latitudes. A very similar bias is found in Chinese astronomy providing another hint at possible links between Indian and Chinese astronomy. The Chinese have traditionally used different values for the eclipse limit north and south of the ecliptic, perhaps to compensate for the southward bias.

Keywords: Eclipse prediction, Indian astronomy, Chinese astronomy, lunar parallax.

1. INTRODUCTION

David Mumford has formulated a statistical framework for testing the accuracy of ancient eclipse predictions [6]. I have adapted his framework and computer code to test the accuracy of eclipse predictions based on Nilakantha’s eclipse theory. Mumford’s formulation tests the accuracy of solar eclipse prediction for randomly chosen mean conjunctions. It involves determination of the time when the true conjunction will be observed at a given location and the distance between the centers of the sun and the moon at that time. A solar eclipse is observed if this distance is less than the sum of the radii of the sun and the moon. Since parallax influences the observed distance between the sun and the moon at a given location, computation of parallax plays a crucial role. Mumford’s formulation is largely a test for the accuracy of parallax
computation. After a brief explanation of parallax, I have outlined a history of parallax computation in India. Results of the statistical analysis of eclipse predictions are described in Section 9. A striking observation is that there are hardly any false negatives. For comparison, predictions of eclipses in Beijing using Nilakantha’s formulas were also analyzed. Again, there are hardly any false negatives, but more importantly, almost all false positives occur south of the ecliptic. Interestingly, eclipses predicted by shoushi li in China show the same pattern. I have included a brief history of parallax computation in China as well.

Geometric astronomy emerged in India in the early first millennium C.E. How it came about is unclear and has been a subject of debate. Its geometric models are based on epicycles, but clearly it is not Ptolemaic. Although influence of pre-Ptolemaic Greek astronomy is evident, details of this contact and its sources remain unknown. The basic framework of Indian astronomy was single epicycle models for the sun and the moon, and models with two epicycles for planets. Menalaus’ spherical trigonometry was apparently unknown in India. Indian astronomers derived the necessary formulas for dealing with spherical geometry from a set of basic planar right triangles aligned with the three coordinate systems (ecliptic, equatorial and horizontal). With the exception of the introduction of lunar evection by Mañjula in the 10th century, this framework remained unchanged from Āryabhaṭa (c. 500 C.E.) to Nilakantha (c. 1500 C.E.). The achievement of the Indian astronomers and the final form it took are perhaps best represented in Nilakantha’s Tantrasaṅgraha [12].

A version of Indian astronomy was brought to China along with Buddhism early in the first millennium. So far, there is no direct evidence that it had much impact on the traditional Chinese approach. Nonetheless, there are notable parallels. Indian astronomy was also introduced in the Islamic world in the eighth century. An influential Islamic text based on Indian astronomy was al-Khwārizmi’s Sindhind [9] (c. 825 C.E.). Ptolemaic astronomy was brought to the Islamic world in the 9th century directly through translations of Greek texts into Arabic, especially the Almagest. Further development in each of these cultures took place according to its own tradition. The early Chinese astronomy relied on polynomial interpolations

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1For example, Āryabhaṭa’s value of 3438′ for the radius in the construction of the table of Rsine is same as the value used by Hipparchus in his chord table. So is the interval between successive values of the angle in the table.
which were later replaced by explicit algebraic formulas. The development of Islamic astronomy was largely Ptolemaic although al-Khwārizmi’s tradition was never fully abandoned [5]. Islamic astronomy used spherical trigonometry extensively. One of its important achievements was the elimination of the equant from the Ptolemaic models. The culmination of the Chinese approach is shoushi lǐ [16] (13th century) while the triumph of Islamic astronomy is perhaps best seen in the works of al-Shāṭir (14th century) and al-Tusi (13th Century).

In the following, I have expressed Indian formulas in modern notation.

2. PARALLAX

The parallax $p$ of a celestial body $P$ when viewed from a point on the surface of the earth is illustrated in Figure 1. The circle represents the surface of the earth. Let $z$ be the zenith distance of $P$ when observed from the center of the earth and let $z'$ be its zenith distance when observed from a point on the surface of the earth. Then,

$$p \approx \sin p = \frac{\rho}{d'} \sin z = \frac{\rho}{d} \sin z'$$

where

$\rho$: the radius of the earth.

$d$: distance of $P$ from the earth’s center.

$d'$ : distance of $P$ from an observer on the surface of the earth.
The horizontal parallax $H$ is the maximum value of the parallax which occurs when $P$ is on the observer’s horizon; that is, when $z' = 90^\circ$; $H = \sin H = \rho/d$.

For predicting the time and magnitude of solar eclipses, it is necessary to calculate the components of $p$ along the ecliptic coordinates, namely, the parallax in longitude and the parallax in latitude. ($p$ is called the parallax in altitude.) In Figure 2, $P$ is geocentrically observed to be on the ecliptic. $P'$ is the position of $P$ as seen by an observer on the surface of the earth. The arc $P'Q$ is the parallax in latitude and the arc $PQ$ is the parallax in longitude.

Let $\gamma$ denote the angle between the ecliptic and the vertical circle through $P$. Then,

Parallax in latitude: $p_\beta = p \sin \gamma$

Parallax in longitude: $p_\lambda = p \cos \gamma$
These are the formulas given by Ptolemy in the Almagest except that he calculates $\gamma$ for the moon by assuming that it is on the ecliptic and hence its value is approximate.

Indian astronomers did not determine $p$ or $\gamma$, but proceeded to calculate $p_\beta$ and $p_\lambda$ directly. Let $V$ denote the nonagesimal, the point where the secondary to the ecliptic through the zenith intersects the ecliptic (see Figure 2). Indian methods for calculating the parallax were based on the following fundamental formulas:

Parallax in latitude: $p_\beta = \frac{d}{d'} \sin z_V$

Parallax in longitude: $p_\lambda = \frac{d}{d'} \sin z_R$

where

$z_V$: the zenith distance of $V$ (= zenith’s ecliptic latitude).

$z_R$: zenith distance of the secondary to the ecliptic through $P$.

These formulas assume that $P$ is on the ecliptic. Since the moon may not be on the ecliptic, Indian astronomers modified the value of $z_V$ for the moon by subtracting the latitude of the moon.

Before Nilakaṇṭha, no distinction was made between $\frac{d}{d'}$ and the horizontal parallax $\frac{d}{d'}$. Nilakaṇṭha gives the traditional version as well as the correct version. By the time of Brahmagupta (7th century), it was recognized that

$\sin z_R = \sin(\lambda_V - \lambda_P) \cos z_V$.

These formulas were apparently used only in India. al-Khwārizmī did introduce Brahmagupta’s approximate version into Islamic astronomy. Islamic parallax tables in al-Khwārizmī’s tradition continued to be composed [3] although the prevailing Islamic traditions were based on Ptolemy’s Almagest.

3. HORIZONTAL PARALLAX OF THE SUN AND THE MOON

Indian astronomers set the value of the horizontal parallax $H$ of any celestial body equal to one fifteenth of its mean daily angular motion. This follows from their assumption that all celestial bodies move with the same linear speed. Since the angular velocity and the horizontal parallax are both inversely proportional to the
distance of the celestial body, the horizontal parallax is proportional to the angular velocity \( v \): \( H = kv \) for some constant \( k \). Sūryasiddhānta [2] gives the moon’s radius as 800 yojanas and the circumference of its orbit as 324000 yojanas giving us the theoretical value of 53° 20” for the horizontal parallax of the moon. The moon’s mean daily sidereal motion is given as 13° 10’ 35” = 790.59’. Therefore,

\[
k = \frac{53.333}{790.59} \approx \frac{1}{15}.
\]

Although Nilakantha includes this formula in his chapter on solar eclipses, he also gives the correct formula for the parallax, namely, \( p = \sin p = \frac{p}{d'} \sin z \).

Since the moon’s angular velocity is about 13.4 times the sun’s angular velocity, horizontal solar parallax according to the Indian tradition is about 4’ which is about 27 times bigger than its true value. Nilakantha also has a similarly large value since his derivation of the distance of the sun from the center of the earth is based on the same assumption, namely, all celestial bodies have the same linear velocity.


The main problem for the Indian astronomers was how to calculate \( \sin z_i \) and \( \sin z_k \).
They knew formulas pertaining to salient points on the ecliptic indicated in Figure 3. In the figure, A is the ascendant (udayalagna) which is the point where the ecliptic intersects the horizon in the east. M is the mid-heaven (madhyalagna, the point where the ecliptic intersects the local meridian). P represents the sun or the moon. (As mentioned above, the zenith distance of the nonagesimal in the case of the moon is modified by subtracting the moon’s latitude.) We use the following notation:

\[ z_P, z_M, z_V : \text{geocentrically observed zenith distances of P, M, V.} \]
\[ \lambda_P, \lambda_A, \lambda_M, \lambda_V : \text{ecliptic longitudes of P, A, M and V.} \]
\[ a : \text{amplitude of the ascendant, the angle between the vertical circles through A and E.} \]
\[ \delta_P : \text{declination of P.} \]
\[ \phi : \text{local terrestrial latitude.} \]
\[ h_P : \text{hour angle of P.} \]

Since the time of Āryabhaṭa and Paitāmahasiddhānta, Indian astronomers knew how to calculate the ecliptic longitudes \( \lambda_P, \lambda_M, \lambda_V \) from the given values of \( \lambda_A \) and \( h_P \) using a table of oblique ascension of the zodiacal signs. They had exact formulas for \( z_M \) and \( \sin a \). They also had an exact formula for \( z_P \) which when simplified takes the following modern form of coordinate transformation from the equatorial to the horizontal:

\[ \cos z_P = \sin \phi \sin \delta_P + \cos \phi \cos \delta_P \cos h_P. \]

Nonetheless, Āryabhaṭa’s school did not succeed in deriving exact formulas for

\(^3\)R.C. Gupta interprets a comment by Nilakaṇṭha in his commentary on Āryabhaṭīya of Āryabhaṭa as saying that Āryabhaṭa’s formulas were correct, but had been misinterpreted. (Gupta, R. C.: Madhava’s rule for finding angle between the ecliptic and the horizon and Āryabhaṭa’s knowledge of it, Ed: G. Swarup, A.K. Bag, K.S. Shukla, History of Oriental Astronomy, Proceedings of an International Astronomical Union Colloquim, No. 91, 1987.) If so, it seems rather strange that the followers of Āryabhaṭa continued to use incorrect formulas for a thousand years until the time of Madhava and Nilakaṇṭha.
sin $z_v$ and sin $z_r$ until Nilakaṇṭha finally solved the problem. Āryabhata derived approximate formulas by treating spherical triangles $ZVM$ and $ZPR$ as planar triangles. (Āryabhata’s Āryabhaṭīya is very brief, but details may be filled in from the commentary and Mahābhāskarīya of Bhāskara I [15] (6th century) who closely follows Āryabhata.) Āryabhata and his followers including Nilakaṇṭha calculate the product $\sin(z_m)\sin(a)$. This in fact equals $\sin |\lambda_m - \lambda_v|$ and they could have calculated the quantity more simply since they already knew $\lambda_v$ and $\lambda_m$. Āryabhata’s formula $\sin z_v = \sqrt{\sin^2(z_M) - \sin^2|\lambda_m - \lambda_V|}$ follows from the triangle $ZVM$ treated as a planar triangle. Finally, the formula $\sin z_R = \sqrt{\sin^2(z_P) - \sin^2(z_V)}$ follows from the triangle $ZPR$ treated as a planar triangle.


Brahmagupta seems to have recognized that the formula for $\cos z_P$ can be applied to any point on the ecliptic if its ecliptic longitude and the hour angle are known. He has just one terse verse (Brāhma-sphuṭasiddhānta, Verse 5.3) describing how to calculate $\cos(z_v)$ by calculating the hour angle of the nonagesimal and then to use the same formula. Bhāskara II (12th century) is more explicit. He instructs us to solve the problem by imagining that the nonagesimal is the sun. Brahmagupta immediately suggests an easy approximation (also mentioned in Paitāmahasiddhānta):

$$z_v = \text{declination of } V \pm \text{local terrestrial latitude } \varphi.$$ This approximation was widely used in India and it is the version that al-Khwārizmī introduced in the early Islamic astronomy. Brahmagupta’s exact formula does not seem to have ever been used. It is also absent from Tantrasaṅgraha.

Strangely, an eminent scholar like David Pingree misinterprets Brahmagupta’s cryptic verse and comments [10]:

“In computing parallax, Brahmagupta makes the same error as does the Paitamah - that is, he regards the zenith distance of the nonagesimal, $ZV$, to be the same as the zenith distance of a body on the meridian. Thus he states that (BSS 5.3): $\sin ZV = \sin \alpha_m = \sin(\varphi \pm d)$”

The relevant portion of Brahmagupta’s verse reads as follows:

$tadudayairvilagnasamam taduditaghaṭikāṣṭacchaṅkustaccarapraṇaith$

“equal to nonagesimal by means of local rising times. its elapsed time (since its rise)
the sine of its altitude by means of ascensional difference.”

Clearly, the verse is a very brief summary of the method used in calculating the zenith distance of the sun described in an earlier chapter.


Besides introducing lunar evection in Indian astronomy, Mañjula also introduces an algebraic approximation for the longitudinal parallax. This is a rare instance of approximation by a polynomial in Indian tradition. Mañjula’s formula may be stated as

$$p_\lambda = \frac{(20 - h)h}{2} \text{ minutes}$$

where \(h\) is expressed in ghatās. (1 day = 60 ghatās.) Besides being uncharacteristic of Indian tradition, another strange aspect of this formula is that it achieves its maximum of 50’ when \(h = 10\). When \(h = 15\), that is, a quarter of a day, \(p_\lambda = 37.5’\). Mañjula’s commentator Śūryadeva Yaśovā attributes this formula to Lalla (8th-9th century C.E.).


Nilakanṭha, being a follower of Āryabhaṭa, strictly adheres to Āryabhaṭa’s framework. He seems also intent on deriving exact formulas. Since the traditional procedure starts with the determination of the longitudes \(\lambda_A\) and \(\lambda_M\) by interpolating a table of oblique ascension, Nilakanṭha proceeds with the task of deriving exact formulas for these quantities in Chapter 3 of Tantrasaṅgraha. He approaches this by first finding an exact formula for \(\sin z_v\) which amounts to the coordinate transformation:

$$\sin z_v = \cos \varepsilon \sin \varphi - \sin \varepsilon \cos \varphi \sin \alpha_Z$$

where

\(\varepsilon\) : obliquity of the ecliptic.

\(\varphi\) : terrestrial latitude.

\(\alpha_Z\) : right ascension of the zenith.
He then gives the formula for determining the distance of the ascendant from $P$:

$$\sin(\lambda_A - \lambda_p) = \frac{\sin \varphi \sin \delta_p + \cos \varphi \cos \delta_p \cos h_p}{\cos z_V}.$$ 

The numerator is just $\cos z_p$. His formula for determining $\lambda_M$ is

$$\cos \lambda_M = \frac{\cos \alpha_c \cos \epsilon}{\sqrt{1 - \cos^2 \alpha_c \sin^2 \epsilon}}.$$ 

In Chapter 5 on solar eclipses, he gives another formula for $\sin z_V$, different from the one in Chapter 3. It is the correct version of Āryabhaṭa’s formula for $\sin z_V$:

$$\sin z_V = \pm \sqrt{\frac{\sin^2 z_M - \sin^2(z_M) \sin^2(a)}{1 - \sin^2(z_M) \sin^2(a)}}.$$ 

8. PARALLAX COMPUTATION IN CHINA

Lunar parallax was considered for the first time in China by Qutan Xida [19] (718 C.E.) who was an Indian astronomer appointed as an “astronomer royal” in the Tang court. His text, jiuzhi li, is based entirely on Indian astronomy. It includes the Indian method of determining the longitude of the nonagesimal and a version of Brahmagupta’s approximate formula for $z_V$. It calculates the parallax in latitude, but consideration of the parallax in longitude is missing. jiuzhili was not officially adopted.

Soon after, in 727 C.E., Yixing [1] composed dayan li which followed the traditional Chinese paradigm and which was officially adopted. Yixing was a Buddhist monk well-versed in Sanskrit, a skilled mathematician and an astronomer. Typically, he provides tables and uses interpolation for calendric computation. He too considers only the parallax in latitude. It is given in the form of the amount of nodal shift caused by the parallax and is called shicha (“ecliptic difference”). A table of first differences is provided to calculate the parallax at the beginning of each qi. (The ecliptic is divided into 24 qi’s in Chinese astronomy.) The values perfectly fit a quadratic polynomial. (See [13] for a detailed analysis.)

A major improvement in computation of the parallax was made by Xu Ang in his xuanming li [18] (822 C.E.) by taking the hour angle into account. He divided parallax computation into 4 parts: Time difference (shicha), North-South difference (qicha), East-West difference (kecha), Plus difference (jiacha). shicha was used to
account for the time difference between the geocentrically observed (true) conjunction and the locally observed (apparent) conjunction. *qicha* corresponds to Yixing’s *shicha*. It is maximum at the two solstices when the hour angle is zero and assumed to decrease linearly to zero at the equinoxes. The hour angle correction is applied so that the value of *qicha* becomes zero at sunrise and sunset. *kecha* is new. It is assumed to be constant when the sun is less than 3 qi’s away from an equinox and it linearly decreases to zero at the solstices. These values are multiplied by the hour angle to obtain the final value of *kecha*. *jiacha* is an adjustment for the local terrestrial latitude, but it was considered to be negligible in practice by *xuanming li*. It was later taken into account in *shoushi li*. The apparent nodal shift due to parallax in latitude is the algebraic sum of *qicha*, *kecha* and *jiacha*. This is an example of interpolation with respect to two variables in Chinese astronomy (in fact, three variables if we also include *jiacha*). *qicha* and *kecha* have complementary behavior. On solstices at midday, *qicha* is maximum while *kecha* is zero. On equinoxes, it is *kecha* which reaches its maximum at sunrise and sunset while *qicha* is zero.

According to Nakayama [8], a new tradition of using algebraic functions instead of interpolation emerged in Chinese astronomy in the eighth century. This new tradition seems to have originated in Central Asia. Nakayama discusses *futian li* which was unofficially composed by Cao Shiwei (780-783 C.E.) as an example of this tradition. He conjectures that Cao Shiwei or his family came from Samarkand. From Nakayama’s description, it does not seem that *futian li* has an explicit algebraic formula. His suggestion is based on the fact that the values in the table of the equation of the center in *futian li* fits a quadratic equation perfectly. If it were based on observed values and interpolation between them, this would not be the case. One could extend this argument to the table of parallax in *dayan li* which also fits a quadratic equation perfectly. This would suggest that the tradition perhaps goes further back. It is interesting that in *xuanming li* which was composed after *futian li*, *kecha* does not fit an algebraic equation. However, *shoushi li* does have plenty of explicit algebraic formulas.

*shoushi li* perfects Xu Ang’s method of calculating the parallax. Its explicit formulas for lunar parallax observed in Beijing are as follows:

\[
shicha : \frac{\tilde{h}(3000 - \tilde{h})}{9600} = t \quad \text{where} \quad \tilde{h} \quad \text{is the time interval between noon and the syzygy,}
\]

expressed in units of one ten-thousandth of a day. The formula is remarkably similar to *Mañjula*’s formula.
qicha: \(4.46 - \frac{\bar{\lambda}^2}{1870} \left(1 - \frac{\bar{h} + \tau}{d/2}\right)\) where \(\bar{\lambda}\) is the longitudinal distance (in du) of the true sun from the solstices, and \(d\) is the time interval between the sunrise and the sunset, expressed in units of one ten-thousandth of a day. (1 \(du = 0.9856^\circ\).

kecha: \(\frac{\bar{\lambda}(A-\bar{\lambda})}{1870} \left(\frac{h + \tau}{2500}\right)\) where \(A\) is a semicircle expressed in du.

jiacha: A constant term = 6.15.

The parallactic shift in the node in du is the algebraic sum of qicha, kecha and jiacha. What is astonishing is that this sum closely resembles Indian formulation as shown by Qu Anjing [11]. An easy way to see this is to start with Nilakantha’s formula:

\[ p_\beta = H \sin z_v = H (\cos \varepsilon \sin \varphi - \sin \varepsilon \cos \varphi \sin \alpha_s) \]

\[ \sin \alpha_s = \sin(\alpha_s + h_s) = \sin \alpha_s \cos h_s + \cos \alpha_s \sin h_s \] where \(\alpha_s\) and \(h_s\) are the right ascension and the hour angle of the sun. If we approximate the right ascension by the corresponding ecliptic longitude, we get

\[ p_\beta = H (\cos \varepsilon \sin \varphi - \sin \varepsilon \cos \varphi (\sin \lambda_s \cos h_s + \cos \lambda_s \sin h_s)). \]

Parallactic nodal shift = \(p_\beta / \tan \iota\) where \(\iota\) is the inclination of the lunar orbit to the ecliptic. With \(H = 57^\circ\), \(\varphi = 39.91^\circ\), \(\varepsilon = 23.44^\circ\), \(\iota = 5.15^\circ\),

nodal shift in du = 6.203 + 3.264 \(\sin \lambda_s \cos h_s + 3.264 \cos \lambda_s \sin h_s\)

which has the same structure as the formula in shoushi li. The first term on the right may be identified with jiacha while the next two terms take the form of the formulas for qicha and kecha if we substitute the following approximations typical in Chinese tradition:

\[ |\sin \lambda_s| = \cos \bar{\lambda} \approx 1 - \left(\frac{\bar{\lambda}}{91.325}\right)^2; \]

\[ |\cos \lambda_s| = \sin \bar{\lambda} \approx \frac{\bar{\lambda}}{91.325} \left(\frac{A}{91.325} - \frac{\bar{\lambda}}{91.325}\right); \]

\[ \cos h_s \approx 1 - \frac{\bar{h} + \tau}{d/2}, \quad \sin h_s \approx \frac{\bar{h} + \tau}{2500} \]

(The number 2500 is in day parts and thus equals a quarter of a day.)
While the value of \( jiacha \) is quite close to the theoretical value, the theoretical maximum of the second and the third term is much smaller than the maximum 4.46 of \( qicha \) and \( kecha \) in \emph{shoushi li}. I think this discrepancy may be reconciled by following the rationale offered by Nakayama [7] and Sivin [17]. According to this rationale, Chinese astronomers first calculated the nodal shift for an observer on the equator. They then assumed that the values of \( qicha \) and \( kecha \) on the equator remain valid for all terrestrial latitudes. The correction due to change in terrestrial latitude was made simply by adding an appropriate constant, namely \( jiacha \). For an observer on the equator, \( jiacha = 0 \) and

\[
\text{nodal shift } H = 4.255 \sin \lambda_s \cos h_s + 4.255 \cos \lambda_s \sin h_s.
\]

The maximum value of each term on the right is 4.255 which is close to the value of 4.46 in \emph{shoushi li}. As usual, \emph{shoushi li} is silent as to how these formulas were derived. It is an interesting question as to how the Chinese astronomers determined the values \( qicha \) and \( kecha \) at the equator.

9. **ACCURACY OF NĪLAKAṆṬHA’S PREDICTION OF SOLAR ECLIPSES**

David Mumford has taken a statistical approach for testing the accuracy of ancient algorithms for prediction of solar eclipses. The emphasis is on testing calculation of the parallax. Algorithms for eclipse prediction are initialized by assigning randomly chosen longitudes to mean conjunction, perigee of the moon and a lunar node, randomly chosen to be ascending or descending. The longitude of the node is restricted to be within 15 degrees of the conjunction. Since the formulas in \emph{shoushi li} are specialized for Beijing, perihelion is assumed to occur at the winter solstice in Beijing at noon. Only the eclipses predicted during daytime are taken into account. In order to discount borderline eclipses, a prediction is considered \textit{strong} if its magnitude is at least 0.1 and the time of the nearest approach is at least one hour after the sunrise and one hour before the sunset. A prediction is \textit{weak} if it is not \textit{strong}. (For more details, see [6].) The results for a particular algorithm are compared with predictions made by modern theory to test its accuracy.

In this paper, I report results for \emph{Tantrasaṅgraha} obtained by adapting Mumford’s statistical approach and his computer code. The first set of results were obtained by applying \emph{Nilakaṇṭha’s} algorithm to predict solar eclipses in Beijing (latitude = 39.91 degrees).
Out of 10000 random trials

- Modern theory predicted strong eclipses in 1310 cases. Out of these, 1308 were predicted as strong eclipses by Tantrasaṅgraha as well. The remaining 2 were predicted as weak eclipses by Tantrasaṅgraha.

- Tantrasaṅgraha predicted 1533 strong eclipses. According to the modern theory, 1308 were correctly strong, but 165 were actually weak and 60 were false positives, that is, no eclipse at all according to the modern theory. There were no false negatives.

- The standard deviation of the error in the time of the maximum eclipse was about 13 minutes.

A striking feature of the 60 false predictions was that 54 occurred south of the ecliptic. In contrast, out of the total of 1535 cases of strong eclipses predicted by either Tantrasaṅgraha or the modern theory, 854 (about 56%) occurred south of the ecliptic, indicating a strong southward bias in the case of false positives.

To further investigate this bias, I ran the comparison at latitudes 0 and -39.91 degrees. On the equator, the number of false predictions by Tantrasaṅgraha north and south of the ecliptic were about equal, but at latitude -39.91, almost all of them occurred north of the ecliptic.

I also analyzed eclipse predictions according to shoushi li. Interestingly, they were found to have a similar bias. This provides a possible explanation as to why the Chinese have traditionally used different values for the eclipse limit north and south of the ecliptic, apparently compensating for the southward bias.

As a further experiment, I ran the trials again correcting the traditional values of some of the Indian astronomical constants.

- Replace the traditional value of the obliquity 24° by 23.439291°.
- Replace the traditional value of the inclination of the lunar orbit 4.5° by 5.145°.
- Indian value of the horizontal solar parallax is about 27 times the true value. Since the solar parallax is very small, simply set it equal to zero.

The result was that the error rate remained about the same, but the southward bias disappeared.
10. CONCLUSION

A brief history of Nilakantha’s parallax computation is traced. The accuracy of his solar predictions is evaluated. Tantrasaṅgraha is found to be remarkably accurate. The algorithm has a pronounced bias towards predicting false positives rather than false negatives paralleling a similar bias in the case of shoushi li. Since predictions of solar eclipses had huge political implications in China, it is reasonable to assume that it was safer for Chinese astronomers to predict an eclipse and risk of being wrong than miss an eclipse. No such political significance was attached to false predictions in India and it is surprising that Indian astronomy has the same bias. More interesting is the fact that most of the false positives predicted for Beijing by both the systems occur south of the ecliptic providing another hint at possible links between Indian and Chinese astronomy. The Chinese method of choosing different values for the eclipse limit north and south of the ecliptic seem to indicate that they were aware of the southward bias. Being close to the equator, Nilakantha would not have noticed this bias.

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