APPENDIX C
Calculus and the TI-85 Calculator

Functions

A. Define a function, say \(y_1\), from the Home screen
- Press \(\text{2nd} \ [\text{QUIT}]\) to invoke the Home screen.
- Press \(\text{2nd} \ [\text{alpha}] \ [Y] \ 1 \ \text{ALPHA} \ [=]\) followed by an expression for the function, and press \(\text{ENTER}\).

B. Define a function from the function editor
- Press \(\text{GRAPH} \ [\text{F}1]\) to select \(y(x) = \) from the GRAPH menu and obtain the screen for defining functions; that is, the function editor.
- Cursor down to a function. (To delete an existing expression, press \(\text{CLEAR}\). To create an additional function, cursor down to the last function and press \(\text{EXIT}\).)
- Type in an expression for the function. (Press \(\text{F}1\) or \(\text{VAR}\) to display \(x\). Press \(\text{F}2\) or \(\text{2nd} \ [\text{alpha}] \ [Y]\) to display \(y\).)

C. Select or deselect a function in the function editor
(Functions with highlighted equal signs are said to be selected. The graph screen displays the graphs of all selected functions.)
- Press \(\text{GRAPH} \ [\text{F}1]\) to invoke the function editor.
- Cursor down to the function to be selected or deselected.
- Press \(\text{F}3\), that is, SELCT, to toggle the state of the function on and off.

D. Display a function name, that is, \(y_1, y_2, y_3, \ldots\)
- Press \(\text{2nd} \ [\text{alpha}] \ [Y]\) followed by the number. or
- Press \(\text{2nd} \ [\text{VARS}] \ [\text{MORE}] \ [\text{F}3]\) to invoke a list containing the function names.
- Cursor down to the desired function.
- Press \(\text{ENTER}\) to display the selected function name.

E. Combine functions
Suppose \(y_1\) is \(f(x)\) and \(y_2\) is \(g(x)\).
- If \(y_3 = y_1 + y_2\), then \(y_3\) is \(f(x) + g(x)\). (Similarly for \(-, \times, \text{and} \div\).)
- If \(y_3 = \text{evalF}(y_1, x, y_2)\), then \(y_3\) is \(f(g(x))\). (To display \(\text{evalF}(\), press \(\text{2nd} \ [\text{CALC}] \ [\text{F}1]\).)

Specify Window Settings

A. Customize a window
- Press \(\text{GRAPH} \ [\text{F}2]\) to invoke the RANGE screen and edit the following values as desired.
- \(x\text{Min} = \) the leftmost value on the \(x\)-axis
- \(x\text{Max} = \) the rightmost value on the \(x\)-axis
- \(x\text{Scl} = \) the distance between tick marks on the \(x\)-axis
- \(y\text{Min} = \) the bottom value on the \(y\)-axis
- \(y\text{Max} = \) the top value on the \(y\)-axis
- \(y\text{Scl} = \) the distance between tick marks on the \(y\)-axis

Note 1: The notation \([a, b]\) by \([c, d]\) stands for the range settings \(x\text{Min} = a, x\text{Max} = b, y\text{Min} = c, y\text{Max} = d\).

Note 2: The default values of \(x\text{Scl}\) and \(y\text{Scl}\) are 1. The value of \(x\text{Scl}\) should be made large (small) if the difference between \(x\text{Max}\) and \(x\text{Min}\) is large (small). For instance, with the window settings \([0, 100]\) by \([-1, 1]\), good scale settings are \(x\text{Scl} = 10\) and \(y\text{Scl} = .1\).

B. Use a predefined range setting
- Press \(\text{GRAPH} \ [\text{F}3]\) to invoke the ZOOM menu.
- Press \(\text{F}1\), that is, ZSTD, to obtain \([-10, 10]\) by \([-10, 10]\), \(x\text{Scl} = y\text{Scl} = 1\).
- Press \(\text{MORE} \ [\text{F}2]\), that is, ZSQR, to obtain a true-aspect viewing rectangle. (With such a viewing rectangle, lines that should be perpendicular actually look perpendicular, and the graph of \(y = \sqrt{1 - x^2}\) looks like the top half of a circle.)
- Press \(\text{MORE} \ [\text{F}3]\), that is ZDECM, to obtain \([-6.3, 6.3]\) by \([-3.1, 3.1]\), \(x\text{Scl} = y\text{Scl} = 1\). (When TRACE is used with this viewing rectangle, points have nice \(x\)-coordinates.)
- Press \(\text{MORE} \ [\text{F}4]\), that is ZTRIG, to obtain \([-21\pi/8, 21\pi/8]\) by \([-4.4]\), \(x\text{Scl} = \pi/2, y\text{Scl} = 1\), a good setting for the graphs of trigonometric functions.

C. Some nice range settings
With these settings, one unit on the \(x\)-axis has the same length as one unit on the \(y\)-axis, and tracing progresses over simple values.
Derivative, Slopes, and Tangent Lines

A. Compute \( f'(a) \) from the Home screen using \( \text{der1}(f(x),a) \)
   - Press \( \text{2nd} \) \( \text{[CALC]} \) \( \text{F}3 \) to display \( \text{der1} \).
   - Enter either \( y_1, y_2, \ldots \) or an expression for \( f(x) \).
   - Type in the remaining items and press \( \text{ENTER} \).

B. Define derivatives of the function \( y_5 \)
   - Set \( y_1 = \text{der1}(y_5,x,x) \) to obtain the 1st derivative.
   - Set \( y_2 = \text{der2}(y_5,x,x) \) to obtain the 2nd derivative.
   - Set \( y_3 = \text{nDer}(y_2,x,x) \) to obtain the 3rd derivative.
   - Set \( y_4 = \text{nDer}(y_3,x,x) \) to obtain the 4th derivative.

   Note: \( \text{der1}, \text{der2}, \) and \( \text{nDer} \) are found on the menu obtained by pressing \( \text{2nd} \) \( \text{[CALC]} \).

C. Compute the slope of a graph at a point
   - Press \( \text{GRAPH} \) \( \text{F}3 \) to display the graph of the function.
   - Press \( \text{MORE} \) \( \text{F}1 \) \( \text{F}2 \) to select \( \text{dy/dx} \) from the \( \text{GRAPH/MATH} \) menu.
   - Use the arrow keys to move to the point of the graph.
   - Press \( \text{ENTER} \).

   Note: This process usually works best with a nice range setting.

D. Draw a tangent line to a graph
   - Press \( \text{GRAPH} \) \( \text{F}3 \) to display the graph of the function.
   - Press \( \text{MORE} \) \( \text{F}1 \) \( \text{MORE} \) \( \text{MORE} \) \( \text{F}3 \) to select \( \text{TANLN} \) from the \( \text{GRAPH/MATH} \) menu.
   - Move the cursor to any point on the graph.
   - Press \( \text{ENTER} \) to draw the tangent line through the point and display the slope of the curve at that point. The slope is displayed at the bottom of the screen as \( \text{dy/dx} = \text{slope} \).

Special Points on the Graph of \( y_1 \)

A. Find a point of intersection with the graph of \( y_2 \), from the Home screen
   - Press \( \text{2nd} \) \( \text{[SOLVE]} \).
   - To the right of \( \text{eqn:} \) enter \( y_1 - y_2 = 0 \) and press \( \text{ENTER} \). \( y_1 \) and \( y_2 \) can be entered via F keys and the equal sign is entered with \( \text{ALPHA} \) \( [=] \).
   - To the right of \( \text{eqn:} \) type in a guess for the \( x \)-coordinate of the point of intersection, and then press \( \text{F}5 \). After a little delay, a value of \( x \) for which \( y_1 = y_2 \) will be displayed in place of your guess.

B. Find intersection points, with graphs displayed
   - Press \( \text{GRAPH} \) \( \text{F}5 \) to display the graphs of all selected functions.
   - Press \( \text{MORE} \) \( \text{F}1 \) \( \text{MORE} \) \( \text{F}5 \) to select ISECT from the \( \text{GRAPH/MATH} \) menu.
   - If necessary, use \( \text{\uparrow} \) or \( \text{\downarrow} \) to place the cursor on one of the two curves.
   - Move the cursor close to the point of intersection and then press \( \text{ENTER} \).
   - If necessary, use \( \text{\uparrow} \) or \( \text{\downarrow} \) to place the cursor on the other curve.
   - Press \( \text{ENTER} \) to display the coordinates of the point of intersection.

C. Find the second coordinate of the point whose first coordinate is \( a \)

From the Home screen
Compute \( \text{evalF}(y_1,a) \) as follows:
   - Press \( \text{2nd} \) \( \text{[CALC]} \) \( \text{F}1 \) to display \( \text{evalF} \).
   - Enter either \( y_1 \) or an expression for the function.
   - Type in the remaining items and press \( \text{ENTER} \) or
   - Press \( \text{a STO} \) \( \text{\text{\textit{\textbf{x-Var}}} \text{ENTER} \) to assign the value \( a \) to the variable \( x \).
   - Display \( y_1 \) and press \( \text{ENTER} \).

From the Home screen or with the graph displayed:
A14 Appendices

- Press **GRAPH** MORE MORE F1.
- Type in the value of \(a\) and press **ENTER**. (The value of \(a\) must be between xMin and xMax.)
- If desired, press the up-arrow key, ▲, to move to points on graphs of other selected functions.

**With the graph displayed:**
- Press F1: that is, TRACE.
- Move cursor with ▼ and/or ▲ until the \(x\)-coordinate of the cursor is as close as possible to \(a\).

**Note:** Usually works best if one of the nice range settings discussed above is used.

D. **Find the first coordinate of a point whose second coordinate is \(b\)**
- Set \(y_2 = b\).
- Find the point of intersection of the graphs of \(y_1\) and \(y_2\) as in part B above.

E. **Find an \(x\)-intercept of a graph of a function**
- Press **GRAPH** MORE F1 F3 to select ROOT from the GRAPH/MATH menu.
- Move the cursor along the graph of the function close to an \(x\)-intercept, and press **ENTER**.

F. **Find a relative extreme point**
- Set \(y_2 = \text{der1}(y_1, x, x)\) or set \(y_2\) equal to the exact expression for the derivative of \(y_1\). [To display \text{der1}, press 2nd [CALC] F3]
- Select \(y_2\) and deselect all other functions.
- Graph \(y_2\).
- Find an \(x\)-intercept of \(y_2\), call it \(r\), at which the graph of \(y_2\) crosses the \(x\)-axis.
- The point \((r, y_1(r))\) will be a possible relative extreme point of \(y_1\).

G. **Find an inflection point**
- Set \(y_2 = \text{der2}(y_1, x, x)\) or set \(y_2\) equal to the exact expression for the second derivative of \(y_1\). (To display \text{der2}, press 2nd [CALC] F4)
- Select \(y_2\) and deselect all other functions.
- Graph \(y_2\).
- Find an \(x\)-intercept of \(y_2\), call it \(r\), at which the graph of \(y_2\) crosses the \(x\)-axis.
- The point whose first coordinate is \(r\) will be a possible inflection point of \(y_1\).

Riemann Sums
Suppose that \(y_1\) is \(f(x)\), and \(c, d\), and \(\Delta x\) are numbers, then \(\text{sum(seq}(y_1, x, c, d, \Delta x))\) computes

\[
f(c) + f(c + \Delta x) + f(c + 2\Delta x) + \cdots + f(d).
\]

The functions \(\text{sum}\) and \(\text{seq}\) are found in the LIST/OPS menu.

- Press 2nd [LIST] F3 MORE F1 F3 F3 to display \text{sum(seq}(\).
- Enter either \(y_1\) or an expression for \(f(x)\).
- Type in the remaining items and press **ENTER**.

Definite Integrals and Antiderivatives

A. **Compute \(\int_a^b f(x)\, dx\)**
On the Home screen, evaluate \(\text{fnInt}(f(x), x, a, b)\) as follows.
- Press 2nd [CALC] F3 to display \(\text{fnInt}\).
- Enter either \(y_1\) or an expression for \(f(x)\).
- Type in the remaining items and press **ENTER**.

B. **Find the area of a region under the graph of a function**
- Press **GRAPH** MORE F1 F3 to select \(\int f(x)\) from the GRAPH/MATH menu.
- If necessary, use ▲ or ▼ to move the cursor to the graph.
- Move the cursor to the left endpoint of the region and press **ENTER**.
- Move the cursor to the right endpoint of the region and press **ENTER**.

**Note:** This process usually works best with a nice range setting.

C. **Obtain the graph of the solution to the differential equation \(y' = g(x), y(a) = b\)**
That is, obtain the graph of the function \(f(x)\) that is an antiderivative of \(g(x)\) and satisfies the additional condition \(f(a) = b\).
- Set \(y_1 = g(x)\).
- Set \(y_2 = \text{fnInt}(y_1, x, a, x) + b\). (To display \(\text{fnInt}(\), press 2nd [CALC] F3) The function \(y_2\) is an antiderivative of \(g(x)\) and can be evaluated and graphed.

**Note:** The graphing of \(y_2\) proceeds very slowly.
D. Shade the region between two curves

Suppose the graph of $y_1$ lies below the graph of $y_2$ for $a \leq x \leq b$ and both functions have been selected. To shade the region between these two curves, execute the instruction $\text{Shade}(y_1,y_2,a,b)$ as follows.

- Press $\text{GRAPH} \ [\text{MORE}] \ [\text{F}2] \ [\text{F}1]$ to display $\text{Shade}$ from the $\text{GRAPH}/\text{DRAW}$ menu.
- Type in the remaining items and press $\text{ENTER}$.

Note: To remove the shading, press $\text{GRAPH} \ [\text{MORE}] \ [\text{F}2] \ [\text{F}1] \ [\text{F}5]$ to execute $\text{ClrDraw}$ from the $\text{MATH}/\text{DRAW}$ menu.

Functions of Several Variables

A. Specify a function of several variables and its derivatives

- In the $y(x) = \text{function editor}$, set $y_1 = f(x,y)$. (The letters $x$ and $y$ can be entered by pressing $\text{F}1$ and $\text{F}2$.)
- Set $y_2 = \text{der1}(y_1,x,x)$. $y_2$ will be $\frac{\partial f}{\partial x}$.
- Set $y_3 = \text{der1}(y_1,y,y)$. $y_3$ will be $\frac{\partial f}{\partial y}$.
- Set $y_4 = \text{der2}(y_1,x,x)$. $y_4$ will be $\frac{\partial^2 f}{\partial x^2}$.
- Set $y_5 = \text{der2}(y_1,y,y)$. $y_5$ will be $\frac{\partial^2 f}{\partial y^2}$.
- Set $y_6 = \text{nDer}(y_3,x,x)$. $y_6$ will be $\frac{\partial^2 f}{\partial x \partial y}$.

B. Evaluate one of the functions in part A at $x = a$ and $y = b$

- On the Home screen, assign the value $a$ to the variable $x$ with $\text{STO} \ [x \ - \ \text{VAR}]$.
- Press $b \ \text{STO} \ [x \ - \ \text{VAR}]$ [2nd] [alpha] $[Y]$ to assign the value $b$ to the variable $y$.
- Display the name of one of the functions, such as $y_1,y_2,...$, and press $\text{ENTER}$.

Least-Squares Approximations

A. Obtain the equation of the least-squares line

Assume the points are $(x_1,y_1), \ldots, (x_n,y_n)$.

- Press $\text{STAT} \ [\text{F}2] \ [\text{ENTER}] \ [\text{ENTER}] \ [\text{F}2]$ to obtain a list used for entering the data.
- Press $\text{F}5$ to clear all previous data.
- Enter the data for the points by pressing $x_1 \ [\text{ENTER}] \ y_1 \ [\text{ENTER}] \ x_2 \ [\text{ENTER}] \ y_2 \ [\text{ENTER}] \ldots \ x_n \ [\text{ENTER}] \ y_n$.

- Press $\text{STAT} \ [\text{F}1] \ [\text{ENTER}] \ [\text{ENTER}] \ [\text{F}2]$ to obtain the values of $a$ and $b$ where the least-squares line has equation $y = bx + a$.

- If desired, the straight line (and the points) can be graphed with the following steps:
  (a) First press $\text{GRAPH} \ [\text{F}1]$ and deselect all functions.
  (b) Press $\text{STAT} \ [\text{F}3]$ to invoke the statistical DRAW menu. (If any graphs appear, press $\text{F}5$ to clear them.)
  (c) Press $\text{F}4$ to draw the least-squares line and press $\text{F}2$ to draw the $n$ points.

B. Assign the least-squares line to a function

- Press $\text{GRAPH} \ [\text{F}1]$ and deselect all functions.
- Press $\text{STAT} \ [\text{F}3] \ [\text{F}2]$ to select SCAT from the STAT/DRAW menu.

Note 1: Make sure the current window setting is large enough to display the points.

Note 2: To also draw the least-squares line, press $\text{STAT} \ [\text{F}3] \ [\text{F}4]$ to select DRREG from the STAT/DRAW menu.

Note 3: To erase the points, press $\text{STAT} \ [\text{F}3] \ [\text{F}5]$ to select CLDRW from the STAT/DRAW menu.

The Differential Equation $y' = g(t,y)$

The TI-85 uses a numerical approximation technique that is different from Euler’s method.

Carry out a numerical approximation with $a$, $b$, $y_0$, $h$ and $n$ as given in Section 10.7

- To invoke differential equation mode, press [2nd] [MODE], move the cursor down to the fifth line, move the cursor right to DifEq, and press $\text{ENTER}$.
- Press $\text{GRAPH} \ [\text{F}1]$ and enter the differential equation. Use Q1 (or Q2, Q3, ...) instead of $y_1$ (or $y_2$, $y_3$, ...). The letters $t$ and $Q$ can be entered with the keys $[\text{F}1]$ and $[\text{F}2]$. Up to nine differential equations, with function variables Q1, Q2, ..., can be specified.
- Press $\text{GRAPH} \ [\text{F}4]$ and set $x = t$ and $y = Q$.
- Press $\text{GRAPH} \ [\text{F}2]$ to invoke the range-setting screen.
• Set tMin and xMin to a, set tMax and xMax to b, and set tStep to h. (Leave the values of tPlot and difTol at their default settings of 0 and .001.)
• Set the values of xScl, yMin, yMax, and yScl as you would when graphing ordinary functions.
• Press \texttt{GRAPH F3} and set the initial value (denoted by QI1, QI2, etc.) to y0.
• Press \texttt{GRAPH F5} to see a graph of a numerical solution of the differential equation.

\textit{Note} 1: To make the graph more accurate (and the graphing slower), decrease the value of tStep.

\textit{Note} 2: You can simultaneously graph a family of solutions to \( Q' = g(t, Q) \) by setting \( Q' = g(t, Q) \), \( Q' = g(t, Q) \), \ldots, and giving each of the differential equations a different initial value.

\textit{Note} 3: When finished with differential equation mode, reset the calculator to function mode by pressing \texttt{2nd \[ mode \]} to display Func in the fifth line, and pressing \texttt{ENTER}.

The Newton–Raphson Algorithm

\textbf{Perform the Newton–Raphson Algorithm}

• Assign the function \( f(x) \) to y1 and the function \( f'(x) \) to y2.
• Press \texttt{2nd \[ quit \]} to invoke the Home screen.
• Type in the initial approximation.
• Assign the value of the approximation to the variable x. This is accomplished with the keystrokes \texttt{STO \[ x \]-VAR \[ enter \].}
• Type in \( x - y1/y2 \rightarrow x \). (This statement calculates the value of \( x - y1/y2 \) and assigns it to x.)
• Press \texttt{ENTER} to display the value of this new approximation. Each time \texttt{ENTER} is pressed, another approximation is displayed.

\textit{Note}: In the first step, \( y2 \) can be set equal to der1(y1,x).x).

Sum a Finite Series

Compute the sum \( f(m) + f(m + 1) + \cdots + f(n) \)

On the Home screen, evaluate \( \texttt{sum(seq(f(x),x,m,n,1)} \) as follows:

• Press \texttt{2nd \[ list \]} \texttt{F5} \texttt{MORE} \texttt{F1} \texttt{F3} to display \texttt{sum(seq).}

• Enter an expression for \( f(x) \).

• Type in the remaining items and press \texttt{ENTER}.

\textit{Note}: About 2500 terms can be summed.

Miscellaneous Items and Tips

A. From the Home screen, if you plan to reuse the most recently entered line with some minor changes, press \texttt{2nd \[ entry \]} to display the previous line. You can then make alterations to the line and press \texttt{ENTER} to execute the line.

B. If you plan to use \texttt{TRACE} to examine the values of various points on a graph, set yMin to a value that is lower than is actually necessary for the graph. Then, the values of \( x \) and \( y \) will not obliterate the graph while you trace.

C. To clear the Home screen, press \texttt{CLEAR} twice.

D. When two menus are displayed at the same time, you can remove the top menu by pressing \texttt{EXIT}. (After that, you can remove the remaining menu with \texttt{CLEAR}).

E. To obtain the solutions of a quadratic equation, or of any equation of the form \( p(x) = 0 \), where \( p(x) \) is a polynomial of degree \( \leq 30 \), press \texttt{2nd \[ poly \]}, enter the degree of the polynomial, enter the coefficients of the polynomial, and press \texttt{F5}. Some of the solutions might be complex numbers.