

## Handout - Fractions and Rational Expressions

### Algebraic Rules for Fractions and Rational Expressions

$a, b, c, d$  may be numbers or variable expressions.

Adding and Subtracting  
requires

a Common Denominator

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \quad \text{has Common Denominator}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{c}{d} \cdot \frac{b}{b} = \frac{a \cdot d + c \cdot b}{b \cdot d}$$

Multiplying  
just multiply tops/bottoms

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

Dividing - "flip" bottom

$$\frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

Reducing Fractions  
cancel common terms

$$\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

$$\frac{a \cdot c + b \cdot c}{d \cdot c} = \frac{a + b}{d}$$

$$\frac{a \cdot c}{b \cdot c + d \cdot c} = \frac{a}{b + d}$$

## Reducing Fractions and Rational Expressions

Technique: **Factor and Cancel** (*Cancel Multiplicative Factors Only*)

Example 1a: Simplify and Reduce:  $\frac{6}{8}$

$$\frac{6}{8} = \frac{2 \cdot 3}{2 \cdot 4}$$

Factor

$$\frac{6}{8} = \frac{\cancel{2} \cdot 3}{\cancel{2} \cdot 4}$$

Cancel common factor

$$= \frac{3}{4}$$

Answer

Example 1b: Simplify and Reduce:  $\frac{7+8}{7+28}$

$$\frac{7+8}{7+28}$$

*cannot cancel 7's!*

$$\frac{7+8}{7+28} = \frac{15}{35}$$

Add

$$= \frac{3 \cdot 5}{5 \cdot 7}$$

Factor

$$= \frac{3 \cdot \cancel{5}}{\cancel{5} \cdot 7}$$

Cancel

$$= \frac{3}{7}$$

Answer

Example 1c: Simplify and Reduce:  $\frac{6x - 9}{6x + 12}$

$\frac{6x - 9}{6x + 12}$	<i>cannot cancel 6x!</i>
$\frac{6x - 9}{6x + 12} = \frac{3 \cdot (2x - 3)}{3 \cdot (2x + 4)}$	Factor
$= \frac{\cancel{3} \cdot (2x - 3)}{\cancel{3} \cdot (2x + 4)}$	Cancel
$= \frac{2x - 3}{2x + 4}$	Answer

Example 1d: Simplify and Reduce:  $\frac{9x + 6}{15x + 10}$

$\frac{9x + 6}{15x + 10} = \frac{3 \cdot (3x + 2)}{5 \cdot (3x + 2)}$	Factor
$= \frac{3 \cdot \cancel{(3x + 2)}}{5 \cdot \cancel{(3x + 2)}}$	Cancel Common Factor
$= \frac{3}{5}$	Answer

Example 1e: ( $x$  terms included) Simplify and Reduce:  $\frac{8x^5 + 6x^4}{10x^5 + 20x^3}$

Observe 1: 2 divides all the integers (8, 6, 10, 20)

Observe 2:  $x^3$  divides all the  $x^n$  terms ( $x^5, x^4, x^5, x^3$ )

$$\begin{aligned} \frac{8x^5 + 6x^4}{10x^5 + 20x^3} &= \frac{2 \cdot (4x^5 + 3x^4)}{2 \cdot (5x^5 + 10x^3)} && \text{Factor Common Integers} \\ &= \frac{\cancel{2} \cdot (4x^5 + 3x^4)}{\cancel{2} \cdot (5x^5 + 10x^3)} && \text{Cancel Common Factor} \\ &= \frac{4x^5 + 3x^4}{5x^5 + 10x^3} \\ &= \frac{x^3 \cdot (4x^2 + 3x)}{x^3 \cdot (5x^2 + 10)} && \text{Factor Common } x^n \\ &= \frac{\cancel{x^3} \cdot (4x^2 + 3x)}{\cancel{x^3} \cdot (5x^2 + 10)} && \text{Cancel Common Factor} \\ &= \frac{4x^2 + 3x}{5x^2 + 10} && \text{Answer} \end{aligned}$$

**Simplify and Reduce the following Fractions/Rational Expressions**

a)  $\frac{48}{18}$

b)  $\frac{48}{30}$

c)  $\frac{156}{247}$

d)  $\frac{114}{102}$

e)  $\frac{30 + 6}{27 + 1}$

f)  $\frac{160 + 5}{173 + 7}$

g)  $\frac{12x + 15}{27x - 9}$

h)  $\frac{24x - 36}{4x - 16}$

i)  $\frac{32x^4 - 24x^3}{20x^4 - 44x^3}$

j)  $\frac{84x^3 - 126x^2}{77x^3 + 63x^2}$

k)  $\frac{45x - 40}{9x - 8}$

l)  $\frac{8x - 3}{8x - 3}$

Answers a)  $\frac{8}{3}$ ; b)  $\frac{8}{5}$ ; c)  $\frac{12}{19}$ ; d)  $\frac{19}{17}$ ; e)  $\frac{9}{7}$ ; f)  $\frac{11}{12}$ ; g)  $\frac{4x+5}{9x-3}$ ; h)  $\frac{6x-9}{x-4}$ ; i)  $\frac{6-8x}{11-5x}$ ; j)  $\frac{6(2x-3)}{11x+9}$ ; k) 5; l) 1;

.....

**Multiplying** just multiply the numerators and denominators

Example 2a: Multiply  $\frac{6}{5} \cdot \frac{3}{8}$

$\frac{6}{5} \cdot \frac{3}{8} = \frac{6 \cdot 3}{5 \cdot 8}$	Multiply tops and bottoms
$= \frac{18}{40}$	
$= \frac{9}{20}$	Reduce - Get Answer

Example 2b: Multiply  $\frac{2}{7x-1} \cdot \frac{2-x}{4}$

$\frac{2}{7x-1} \cdot \frac{2-x}{4} = \frac{2 \cdot (2-x)}{(7x-1) \cdot 4}$	Multiply tops and bottoms
$= \frac{4-2x}{28x-4}$	
$= \frac{2-x}{14x-4}$	Reduce - Get Answer

.....

**Dividing** Flip the bottom fraction - then multiply

Example 3a: Divide  $\frac{5/9}{6/7}$

$\frac{5/9}{6/7} = \frac{5}{9} \cdot \frac{7}{6}$	Flip the bottom
$= \frac{5 \cdot 7}{9 \cdot 6}$	
$= \frac{35}{54}$	Answer

Example 3b: Divide  $\frac{x/(x-3)}{(x-3)/(2x+5)}$

$\frac{x/(x-3)}{(x-3)/(2x+5)} = \frac{x}{x-3} \cdot \frac{2x+5}{x-3}$	Flip the bottom
$=$	
$= \frac{2x^2 + 5x}{(x-3)^2}$	Answer

.....

**Adding/Subtracting** Common Denominator - add the numerators.

Example 4a: Add  $\frac{5}{8} + \frac{1}{8}$

$\frac{5}{8} + \frac{1}{8} = \frac{5+1}{8}$	Common Denominator
$= \frac{6}{8}$	
$= \frac{3}{4}$	Reduce - Get Answer

Example 4b: Add  $\frac{19+2x}{3x+9} + \frac{x-1}{3x+9}$

$\frac{19+2x}{3x+9} + \frac{x-1}{3x+9} = \frac{19+2x+x-1}{3x+9}$	Common Denominator
$= \frac{18+3x}{3x+9}$	
$= \frac{x+6}{x+3}$	Reduce - Get Answer



## Arithmetic for Fractions and Rational Expression

### Adding/Subtracting Different Denominators - Get Common Denominator

Example 4c: Add  $\frac{2}{3} + \frac{5}{2}$

$$\begin{aligned}\frac{2}{3} + \frac{5}{2} &= \frac{2}{3} \cdot \frac{2}{2} + \frac{5}{2} \cdot \frac{3}{3} && \text{Make Common Denominator} \\ &= \frac{4}{6} + \frac{15}{6} \\ &= \frac{19}{6} && \text{Answer}\end{aligned}$$

Example 4d Add  $\frac{2}{x+3} + \frac{5x}{x+2}$

$$\begin{aligned}\frac{2}{x+3} + \frac{5x}{x+2} &= \frac{2}{x+3} \cdot \frac{x+2}{x+2} + \frac{5x}{x+2} \cdot \frac{x+3}{x+3} && \text{Get Com. Den.} \\ &= \frac{2x+4}{(x+3)(x+2)} + \frac{5x^2+15}{(x+3)(x+2)} \\ &= \frac{5x^2+17x+4}{(x+3)(x+2)} = \frac{5x^2+17x+4}{x^2+5x+6} && \text{Answer}\end{aligned}$$

**Alternate Formula**  $\frac{\text{Cross Multiply on Top}}{\text{Product of Denominators on Bottom}}: \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + c \cdot b}{b \cdot d}$

## Adding and Subtracting

$$\text{a) } \frac{1}{7} + \frac{6}{17}$$

$$\text{b) } \frac{4}{19} + \frac{4}{7}$$

$$\text{c) } \frac{12}{7} - \frac{15}{2}$$

$$\text{d) } \frac{14}{5} - \frac{10}{3}$$

$$\text{e) } \frac{15}{4} + \frac{1}{4} - \frac{45}{20}$$

$$\text{f) } \frac{15}{4} + \frac{13}{4} - \frac{168}{32}$$

$$\text{g) } \frac{4}{5} + \frac{4}{2+x}$$

$$\text{h) } \frac{1}{5} + \frac{3}{3+x}$$

$$\text{i) } \frac{1}{x+3} + \frac{1}{-5+x}$$

$$\text{j) } \frac{2}{x+3} + \frac{5}{-6+x}$$

Answers a)  $\frac{59}{119}$ ; b)  $\frac{104}{133}$ ; c)  $-\frac{81}{14}$ ; d)  $-\frac{8}{15}$ ; e)  $\frac{7}{4}$ ; f)  $\frac{7}{4}$ ; g)  $\frac{4(x+7)}{5(x+2)}$ ; h)  $\frac{x+18}{5(x+3)}$ ; i)  $\frac{2(x-1)}{(x-5)(x+3)} = \frac{2x-2}{x^2-2x-15}$ ; j)  $\frac{7x+3}{(x-6)(x+3)} = \frac{7x+3}{x^2-3x-18}$ ;

## Multiplying

$$\text{a) } \left(\frac{6}{5} + \frac{5}{2}\right) \cdot \left(\frac{7}{2} - \frac{3}{5}\right)$$

$$\text{b) } \left(\frac{8}{5} + \frac{4}{11}\right) \cdot \left(\frac{8}{11} - \frac{7}{5}\right)$$

$$\text{c) } \left(x + \frac{8}{11}\right) \cdot \left(\frac{9}{11} - x\right)$$

$$\text{d) } \left(x + \frac{6}{7}\right) \cdot \left(\frac{1}{7} - x\right)$$

## More Complicated Fractions

$$\text{a) } \frac{4}{7} + \frac{5}{2 + \frac{1}{7}}$$

$$\text{b) } \frac{4}{7} + \frac{4}{2 + \frac{4}{3}}$$

$$\text{c) } \frac{5}{4} + \frac{5}{1 + \frac{1}{x+3}}$$

$$\text{d) } \frac{5}{3} + \frac{2}{1 + \frac{1}{x+5}}$$

$$\text{e) } \frac{8 + \frac{7}{6}}{\frac{4}{13} + 7}$$

$$\text{f) } \frac{7 + \frac{11}{4}}{\frac{10}{13} + 2}$$

$$\text{g) } \frac{6 + \frac{5}{x}}{\frac{4}{x} + 6}$$

$$\text{h) } \frac{2 + \frac{3}{x}}{\frac{4}{x} + 3}$$

Answers a)  $\frac{1073}{100}$ ; b)  $-\frac{3996}{3025}$ ; c)  $-x^2 + \frac{x}{11} + \frac{72}{121}$ ; d)  $-x^2 - \frac{5x}{7} + \frac{6}{49}$ ;

Answers a)  $\frac{61}{21}$ ; b)  $\frac{62}{35}$ ; c)  $\frac{5(5x+16)}{4(x+4)}$ ; d)  $\frac{11x+60}{3(x+6)}$ ; e)  $\frac{143}{114}$ ; f)  $\frac{169}{48}$ ; g)  $\frac{6x+5}{2(3x+2)}$ ; h)  $\frac{2x+3}{3x+4}$ ;

Don't forget to reduce the fractions.

$$\text{a) } \frac{2}{3} - \frac{8}{11} + \frac{134}{33}$$

$$\text{b) } \frac{2}{11} - \frac{7}{5} + \frac{82}{55}$$

$$\text{c) } \frac{7}{z+3} + \frac{2}{z-3} + \frac{z^2 - 9z + 6}{(z-3)(z+3)}$$

$$\text{d) } \frac{5}{z+7} + \frac{5}{z-5} + \frac{z^2 - 13z - 20}{(z-5)(z+7)}$$

Fractions in the Exponent

$$\text{a) } \frac{x^{17/5}}{\sqrt[5]{x^4}} \cdot (x^{-238})^{1/70}$$

$$\text{b) } \frac{x^{20/3}}{\sqrt[3]{x^6}} \cdot (x^{-85})^{1/15}$$

$$\text{c) } \frac{x^{19/3}}{\sqrt[3]{x^5}} \cdot (x^{-126})^{1/21}$$

$$\text{d) } \frac{x^{19/5}}{\sqrt[5]{x^9}} \cdot (x^{-102})^{1/30}$$

Answers a) 4; b)  $\frac{3}{11}$ ; c) 1; d)  $\frac{z+2}{z+7}$ ;

Answers a)  $\frac{1}{x^{4/5}}$ ; b)  $\frac{1}{x}$ ; c)  $\frac{1}{x^{4/3}}$ ; d)  $\frac{1}{x^{7/5}}$ ;