

Formulas for Exponent and Radicals

Algebraic Rules for Manipulating Exponential and Radicals Expressions.

In the following, n, m, k, j are arbitrary -

they can be integers or rationals or real numbers.

$$\frac{b^n \cdot b^m}{b^k} = b^{n+m-k}$$

Add exponents in the numerator and
Subtract exponent in denominator.

$$\left(\frac{a^n \cdot b^m}{c^k}\right)^j = \frac{a^{n \cdot j} \cdot b^{m \cdot j}}{c^{k \cdot j}}$$

The exponent outside the parentheses
Multiplies the exponents inside.

$$\left(\frac{a^n}{b^m}\right)^{-1} = \frac{b^m}{a^n}$$

Negative exponent "flips" a fraction.

$$b^0 = 1 \quad b = b^1$$

Don't forget these

Convert Radicals to Exponent notation

$$\begin{aligned}\sqrt{a} &= a^{1/2} \\ \sqrt[m]{a} &= a^{1/m} \\ \sqrt[m]{a^n} &= a^{n/m}\end{aligned}$$

Radicals - Reducing

$$\begin{aligned}\sqrt{a^2 \cdot b} &= a\sqrt{b} \\ \sqrt[m]{a^m \cdot b} &= a\sqrt[m]{b}\end{aligned}$$

Remove squares from inside

Exponent and Radicals - Solving Equations

$$x^{n/m} = y \Leftrightarrow x = y^{m/n}$$

Solve a power by a root

Solve a root by a power

Example

a) Simplify $\left(\frac{2}{5}\right)^3$

Method $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5} = \frac{\mathbf{8}}{\mathbf{125}}$

b) Simplify $\left(\frac{2 \cdot 3^2}{5^3}\right)^2$

Method $\left(\frac{2 \cdot 3^2}{5^3}\right)^2 = \frac{2^2 \cdot 3^{2 \cdot 2}}{5^{3 \cdot 2}} = \frac{4 \cdot 81}{15,625} = \frac{\mathbf{324}}{\mathbf{15,625}}$

Illustration: where is the negative?

c) Simplify $(-3)^4$ (the 'negative' is inside the parentheses)

Method $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = \mathbf{81}$

d) Simplify $-(3)^4$ (the 'negative' is outside the parentheses)

Method $-(3)^4 = -(3) \cdot (3) \cdot (3) \cdot (3) = \mathbf{-81}$

e) Simplify $\left(\frac{2}{5}\right)^{-3}$ (the 'negative' is in the exponent)

Method $\left(\frac{2}{5}\right)^{-3} = \frac{1}{(2/5)^3} = (or = \left(\frac{5}{2}\right)^3) = \frac{5^3}{2^3}$
 $= \frac{\mathbf{125}}{\mathbf{8}}$

More Examples

f) Simplify $\frac{x^3 \cdot x^7}{x^5}$

Method $\frac{x^3 \cdot x^7}{x^5} = x^{3+7-5} = \mathbf{x^5}$

g) Simplify $(2a^3b^2)(3ab^4)^3$

Method $(2a^3b^2)(3ab^4)^3 = 2a^3b^2 \cdot 3^3a^3b^{4 \cdot 3}$
 $= (2 \cdot 27)(a^{3+3})(b^{2+12})$
 $= \mathbf{54a^6b^{14}}$

h) Simplify $\left(\frac{x}{y}\right)^3 \left(\frac{y^2x}{z}\right)^4$ (*give answer with only positive exponents*)

Method $\left(\frac{x}{y}\right)^3 \left(\frac{y^2x}{z}\right)^4 = \frac{x^3}{y^3} \cdot \frac{y^{2 \cdot 4}x^4}{z^4}$
 $= \frac{x^{3+4}y^{8-3}}{z^4} = \frac{\mathbf{x^7y^5}}{\mathbf{z^4}}$

More Examples with negatives

i) Simplify $\frac{6st^{-4}}{2s^{-2}t^2}$ (give answer with only positive exponents)

Negative exponents flip location: A negative exponent in the numerator moves to the denominator. And a negative exponent in the denominator moves to the numerator.

Method $\frac{6st^{-4}}{2s^{-2}t^2} = \frac{6ss^2}{2t^4t^2} = \frac{3s^3}{t^6}$

j) Simplify $\left(\frac{y}{3z^3}\right)^{-2}$ (give answer with only positive exponents)

A Negative exponent 'flips' the fraction.

Method $\left(\frac{y}{3z^3}\right)^{-2} = \left(\frac{3z^3}{y}\right)^2 = \frac{9z^6}{y^2}$

More Examples

k) Simplify $\frac{(2x^3)^2(3x^4)}{(x^3)^4}$

Method $\frac{(2x^3)^2(3x^4)}{(x^3)^4} = \frac{2^2x^{3 \cdot 2} \cdot 3x^4}{x^{3 \cdot 4}}$

$$= \frac{(4 \cdot 3)x^6x^4}{x^{12}} = 12x^{6+4-12} = \frac{\mathbf{12}}{\mathbf{x^2}}$$

Examples Simplifying Roots

a) Simplify $\sqrt{8}$

Method $\sqrt{8} = \sqrt{4 \cdot 2} = \mathbf{2\sqrt{2}}$

b) Simplify $\sqrt{75}$

Method $\sqrt{75} = \sqrt{25 \cdot 3} = \mathbf{5\sqrt{3}}$

c) Simplify $\sqrt[3]{x^4}$

Method $\sqrt[3]{x^4} = \sqrt[3]{x^3 \cdot x} = \mathbf{x\sqrt[3]{x}}$

d) Simplify $\sqrt[4]{81x^8y^4}$

Method $\sqrt[4]{81x^8y^4} = \sqrt[4]{81}\sqrt[4]{x^8}\sqrt[4]{y^4} = \mathbf{3x^2y}$

Digression: Technically $\sqrt{x^2} = |x|$ and $\sqrt[4]{x^4} = |x|$ but we will not worry about that at this time.

More Examples

e) Simplify $\sqrt{32} + \sqrt{200}$

$$\begin{aligned}\text{Method} \quad \sqrt{32} + \sqrt{200} &= \sqrt{16 \cdot 2} + \sqrt{100 \cdot 2} \\ &= 4\sqrt{2} + 10\sqrt{2} = \mathbf{14\sqrt{2}}\end{aligned}$$

f) Simplify $\sqrt{25b} - \sqrt{b^3}$

$$\begin{aligned}\text{Method} \quad \sqrt{25b} - \sqrt{b^3} &= \sqrt{25 \cdot b} + \sqrt{b^2 \cdot b} \\ &= 5\sqrt{b} - b\sqrt{b} = \mathbf{(5 - b)\sqrt{b}}\end{aligned}$$

Exponents and Radicals

Evaluate the Expression (*negative exponents*) - without using a calculator

a) -2^{-2}

b) $(-2)^{-2}$

c) $\frac{1}{2^{-3}}$

d) $\frac{3^{-1}}{2^3}$

e) $6^{-1} + 5^{-1}$

f) $-1^{-1} \cdot (-2)^{-2}$

Simplify each Expression (*integer exponents*)

a) $(-3x^2y^3)(2x^9y^8)$

b) $(-6a^7b^4)(3a^3b^5)$

c) $x^2x^4 + x^3x^3$

d) $(-2b^2)(3b^3) + (5b^3)(-3b^2)$

e) $(-m^2)(-m) - m(-m) + m(3m^2)$

f) $(z^2)(-z) - (-z) - z(-z^2) + z(2z)$

Answers a) $-1/4$; b) $1/4$; c) 8 ; d) $1/24$; e) $11/30$; f) $-1/4$;

Answers a) $-6x^{11}y^{11}$; b) $-18a^{10}b^9$ c) $2x^6$; d) $-21b^5$; e) $4m^3 + m^2$; f) $2z^2 + z$

Simplify each Expression - Write answers Without Negative Exponents

a) $\left(\frac{1}{2}x^{-4}y^3\right)\left(\frac{1}{3}x^4y^{-6}\right)$

b) $\left(\frac{1}{3}a^{-5}b\right)(a^4b^{-1})$

c) $\frac{-3m^{-1}n}{-6m^{-1}n^{-1}}$

d) $\frac{-p^{-1}q^{-1}}{-3pq^{-3}}$

e) $(2a^2)^3 + (-3a^3)^2$

f) $(b^{-4})^2 - (-b^{-2})^4$

g) $\left(\frac{6xy^2}{8x^{-4}y^3}\right)^{-3}$

h) $\left(-\frac{15x^{-2}y^9}{18x^2y^3}\right)^{-2}$

Simplify each Expression (*variable exponents*)

a) $(x^{b-1})^3(x^{b-4})^{-2}$

b) $(a^2)^{m+2}(a^3)^{4m}$

c) $(-5a^{2t}b^{-3t})^3$

d) $\frac{-9x^{3w}y^{9v}}{6x^{8w}y^{3v}}$

e) $\left(\frac{a^{s+2}}{a^{2s-3}}\right)^4$

f) $\left(\frac{x^{2a-3}}{x^{-4a+1}}\right)^{-4}$

Answers a) $\frac{1}{6y^3}$; b) $\frac{1}{3a}$; c) $\frac{n^2}{2}$; d) $\frac{q^2}{3p^2}$; e) $17a^6$; f) 0; g) $\frac{64y^3}{27x^{15}}$ h) $\frac{36x^8}{25y^{12}}$

Answers a) x^{b+5} ; b) a^{14m+4} ; c) $\frac{-125a^{6t}}{b^{9t}}$; d) $\frac{-3y^{6v}}{2x^{5w}}$; e) a^{-4s+20} ; f) $x^{-24a+16}$

Radicals Simplify without a calculator - then check using a calculator.

a) $-9^{1/2}$

b) $(-27)^{4/3}$

c) $8^{-4/3}$

d) $\left(\frac{4}{9}\right)^{3/2}$

e) $\sqrt[4]{16^3}$

f) $\sqrt[3]{8^5}$

Simplify each expression (*ignore absolute value at this time.*)

a) $(a^{15})^{1/5}$

b) $(x^6)^{1/6}$

c) $(x^3y^6)^{1/3}$

d) $(16x^4y^8)^{1/4}$

Simplify each expression - write answers without negative exponents.

a) $\frac{6a^{1/2}}{2a^{1/3}}$

b) $\frac{-4y}{2y^{2/3}}$

c) $(a^2b^{1/2})(a^{1/3}b^{-1/2})$

d) $\left(\frac{x^{1/2}y}{y^{1/2}}\right)^3$

Answers a) -3 ; b) 81 ; c) $1/16$; d) $8/27$; e) 8 ; f) 32

Answers a) a^3 ; b) $|x|$; c) xy^2 ; d) $2|x|y^2$;

Answers a) $3a^{1/6}$; b) $-2y^{1/3}$; c) $a^{7/3}$; d) $x^{3/2}y^{3/2}$;

Change Notation radical-exponent - use only positive exponents

a) $a(b^4 + 1)^{-1/2}$

b) $-2^{3/4}$

c) $\sqrt[5]{x^3}$

d) $\sqrt[3]{x^3 + y^3}$

Simplify each radical expression (*assume everything is positive.*)

a) $\sqrt{16x^2}$

b) $\sqrt{\frac{xy}{100}}$

c) $\sqrt[3]{\frac{-8a^3}{b^{15}}}$

d) $\sqrt[4]{\frac{16t^4}{y^8}}$

Rationalize the denominator

a) $\frac{1}{\sqrt{10}}$

b) $\sqrt{\frac{5}{12}}$

c) $\frac{2}{\sqrt[3]{x}}$

d) $\frac{x}{y^{2/5}}$

e) $\frac{2}{5 - \sqrt{6}}$

f) $\frac{1}{\sqrt{x} - \sqrt{y}}$

Answers a) $\frac{a}{\sqrt{b^4+1}}$; b) $-\sqrt[4]{2^3}$; c) $x^{3/5}$; d) $(x^3 + y^3)^{1/3}$;

Answers a) $4x$; b) $\frac{\sqrt{xy}}{10}$; c) $\frac{-2a}{b^5}$; d) $\frac{2t}{y^2}$;

Answers a) $\frac{\sqrt{10}}{10}$; b) $\frac{\sqrt{15}}{6}$; c) $\frac{2\sqrt[3]{x^2}}{x}$; d) $\frac{xy^{3/5}}{y}$; e) $\frac{10+2\sqrt{6}}{19}$; f) $\frac{\sqrt{x}+\sqrt{y}}{x-y}$;

Reduce the radical expression.

a) $\sqrt{28}$

b) $\sqrt[3]{40}$

c) $\sqrt[3]{-250x^4}$

d) $\sqrt[3]{-24a^5}$

Simplify

a) $(-2\sqrt{3})(5\sqrt{6})$

b) $(-3\sqrt{2})(-2\sqrt{3})$

c) $\sqrt{20x^3} + \sqrt{45x^3}$

d) $\sqrt[3]{16a^4} + \sqrt[3]{54a^4}$

Reduce and Rewrite each expression using a single radical sign.

a) $\sqrt[3]{3} \cdot \sqrt{2}$

b) $\sqrt{5} \cdot \sqrt[3]{4}$

c) $\sqrt[3]{\sqrt{7}}$

d) $\sqrt[3]{\sqrt[3]{2a}}$

Answers a) $2\sqrt{7}$; b) $2\sqrt[3]{5}$; c) $-5x\sqrt[3]{2x}$; d) $-2a\sqrt[3]{3a^2}$;

Answers a) $-30\sqrt{2}$; b) $6\sqrt{6}$; c) $5x\sqrt{5x}$; d) $5a\sqrt[3]{2a}$;

Answers a) $\sqrt[6]{72}$; b) $\sqrt[6]{2000}$; c) $\sqrt[6]{7}$; d) $\sqrt[9]{2a}$;

Solve the rational exponent problems for x .

a) $4x^{1/3} = 20$

b) $3x^{1/4} = 15$

c) $3x^{3/4} = 24$

d) $4x^{1/3} + 20 = 0$

Solve the rational exponent problems for x .

a) $x^{4/3} - 16 = 0$

b) $3x^{5/3} + 96 = 0$

c) $(x - 3)^{3/2} = 27$

d) $(x - 7)^4 = 16$

Answers a) 125; b) 625; c) 16; d) -125

Answers a) ± 8 ; b) -8; c) $x = 12$; d) 9, 5

More Practice

1a) Simplify the expression: $(3t^4x^4)(8t^7x^3)$

1b) Solve for x : $5x^{5/3} + 60 = 38940$

2a. Simplify the exponential expression: $\frac{(x^{11}y^{19})^5 \cdot (x^{10}y^5)^{-19}}{x^{-137}y^4}$

2b. Evaluate the expression at $x = 22, y = 11$: $\frac{(x^{11}y^{19})^5 \cdot (x^{10}y^5)^{-19}}{x^{-137}y^4}$

3. Simplify the variable exponential expression: $(x^{t-3})^2(x^{2t-2})^2$

4. Rationalize the denominator: $\frac{4 + 4\sqrt{3}}{5 - \sqrt{3}}$

5. Simplify the radical expression: $14\sqrt{320}$

6a. Simplify the radical expression: $\sqrt{245} + \sqrt{45}$

6b. Simplify the radical expression: $\sqrt{80x^{10}} + \sqrt{180x^{10}}$

Answers

1a) $24t^{11}x^7$ 1b) $6^3 = 216$ 2 a) $\frac{x^2}{y^4}$; b) $\frac{4}{121}$ 3) x^{6t-10} 4) $\frac{32+24\sqrt{3}}{22} = \frac{1}{22}(32 + 24\sqrt{3})$ 5) $112\sqrt{5}$
6a) $10\sqrt{5}$ 6b) $10x^5\sqrt{5}$