Handout - Derivative - Power Rule

Power - First Rules a,b are constants.

Function	Derivative	
$y = f(x)$	$\frac{dy}{dx} = f'(x)$	Notation
	$\frac{dy}{dx}\Big\vert_{x=\#} = f'(\#)$	Means Plug # into derivative
$y = a \cdot x^n$	$\frac{dy}{dx} = a \cdot n \cdot x^{n-1}$	**Power Rule**
$y = a \cdot x$	$\frac{dy}{dx} = a$	$n = 1$ in power rule
$y = a$	$\frac{dy}{dx} = 0$	$n = 0$ in power rule
$y = ax^n + bx^m$	$\frac{dy}{dx} = a \cdot n \cdot x^{n-1} + b \cdot m \cdot x^{m-1}$	**Summation Rule**

Recall: Exponential Change of Notations

Function	Exponential Form	
$\sqrt[n]{x}$	$x^{1/n}$	root becomes fraction
$\frac{1}{x^n}$	x^{-n}	denominator becomes negative

Examples - Evaluating a Derivative

Ex1. Evaluate $f'(-1)$ for $f(x) = 11x^5$

Answer: **55**

$$a = 11 \quad \text{and} \quad n = 5$$

$$\Rightarrow f'(x) = 11 \cdot 5 \cdot x^{5-1} = 55x^4$$

$$\Rightarrow f'(-1) = 55 \cdot (-1)^4$$

Ex2. Evaluate $f'(\frac{1}{5})$ for $f(x) = 7x^4$

Answer: $\mathbf{\frac{28}{125}}$

$$a = 7 \quad \text{and} \quad n = 4$$

$$\Rightarrow f'(x) = 7 \cdot 4 \cdot x^{4-1} = 28x^3$$

$$\Rightarrow f'(\tfrac{1}{5}) = 28 \cdot (\tfrac{1}{5})^3$$

Ex3. Evaluate $f'(-\frac{3}{4})$ for $f(x) = 5x^2$

Answer: $\mathbf{-\frac{15}{2}}$

$$a = 5 \quad \text{and} \quad n = 2$$

$$\Rightarrow f'(x) = 5 \cdot 2 \cdot x^{2-1} = 10x$$

$$\Rightarrow f'(-\tfrac{3}{4}) = 10 \cdot (-\tfrac{3}{4})^1$$

Ex4. Evaluate $f'(3)$ for $f(x) = x^6$

Answer: **1458**

$$a = 1 \quad \text{and} \quad n = 6$$

$$\Rightarrow f'(x) = 1 \cdot 6 \cdot x^{6-1} = 6x^5$$

$$Evaluate\ at\ x = 3$$

More Examples

Function	Derivative
$y = 5 \cdot x^7$	$\frac{dy}{dx} = 5 \cdot 7 \cdot x^{7-1} = 35x^6$
$y = 9 \cdot \sqrt[3]{x^2}$ *first change to exponential notation* $y = 9 \cdot x^{2/3}$	$\frac{dy}{dx} = 9 \cdot \frac{2}{3} \cdot x^{2/3-1} = 6x^{-1/3} = \frac{6}{\sqrt[3]{x}}$
$y = \frac{3x^6}{4} + \frac{2}{3x} - 9$ *change to exponential notation* $y = \frac{3}{4}x^6 + \frac{2}{3}x^{-1} - 9$	$\frac{dy}{dx} = \frac{3}{4} \cdot 6 \cdot x^{6-1} + \frac{2}{3}(-1)x^{-1-1} - 0$ $\frac{dy}{dx} = \frac{9}{2} \cdot x^5 - \frac{2}{3}x^{-2}$ $\frac{dy}{dx} = \frac{9x^5}{2} - \frac{2}{3x^2}$

Calculate the derivative and Evaluate at the indicated value of x.

a) Evaluate $f'(3)$ for $f(x) = 5x^4$

b) Evaluate $f'(0)$ for $f(x) = 10x^3$

c) Evaluate $f'(-3)$ for $f(x) = 3x^3$

d) Evaluate $f'(0)$ for $f(x) = x^3$

e) Evaluate $f'(\frac{4}{3})$ for $f(x) = 10x^2$

f) Evaluate $f'(\frac{7}{4})$ for $f(x) = 9x$

g) Evaluate $f'(\frac{7}{4})$ for $f(x) = 2x$

h) Evaluate $f'(-\frac{5}{2})$ for $f(x) = 11x^3$

i) Evaluate $f'(2)$ for $f(x) = \dfrac{12x^{9/2}}{5}$

j) Evaluate $f'(2)$ for $f(x) = \dfrac{3x^2}{2}$

k) Evaluate $f'(3)$ for $f(x) = x^{4/7}$

l) Evaluate $f'(1)$ for $f(x) = \dfrac{12x^{3/5}}{7}$

Answers a) 540; b) 0; c) 81; d) 0;

e) $\frac{80}{3}$; f) 9; g) 2; h) $\frac{825}{4}$;

i) $\frac{432\sqrt{2}}{5}$; j) 6; k) $\frac{4}{7\ 3^{3/7}}$; l) $\frac{36}{35}$;

Calculate the derivative with respect to the independent variable

a) $k(x) = \dfrac{5x^9}{8} + 7x^3 - 17x$

b) $f(v) = \dfrac{9v^9}{5} + v^2 - 11v$

c) $h(z) = -\dfrac{14}{\sqrt{z}} - z^{10/3} + \dfrac{7z^{19/2}}{6}$

d) $h(y) = -\dfrac{19}{\sqrt{y}} + 5y^{10/3} + \dfrac{3y^{17/2}}{2}$

Calculate the following

a) $\left.\dfrac{dy}{dt}\right|_{t=1}$ for $y = t^2 + t + 10$

b) $\left.\dfrac{dy}{dx}\right|_{x=1}$ for $y = x^3 + 8x + 17$

c) $\left.\dfrac{dr}{dz}\right|_{z=0}$ for $r = 4z^2 - 7z - 12$

d) $\left.\dfrac{dv}{ds}\right|_{s=1}$ for $v = s^2 - 8s + 8$

Find the equation of the tangent line ($y = mx + b$) at the indicated point.

a) $y = x^2 - 2x + 5$ at $x = -1$

b) $y = 2x^2 - 4x + 2$ at $x = 1$

c) $y = x^2 + 5x + 8$ at $x = 2$

d)$y = 3x^2 - 8x + 4$ at $x = -3$

Answers a) $k'(x) = \frac{45x^8}{8} + 21x^2 - 17$; b) $f'(v) = \frac{81v^8}{5} + 2v - 11$;
c) $h'(z) = \frac{7}{z^{3/2}} - \frac{10z^{7/3}}{3} + \frac{133z^{17/2}}{12}$; d) $h'(y) = \frac{19}{2y^{3/2}} + \frac{50y^{7/3}}{3} + \frac{51y^{15/2}}{4}$;

Answers a) 3, using $y' = 2t + 1$ at $t = 1$; b) 11, using $y' = 3x^2 + 8$ at $x = 1$;
c) -7, using $r' = 8z - 7$ at $z = 0$; d) -6, using $v' = 2s - 8$ at $s = 1$;

Answers a) $y = 4 - 4x$; b) $y = 0$; c) $y = 9x + 4$; d) $y = -26x - 23$;