**Fractions and Rational Expressions**

Algebraic Rules for Fractions and Rational Expressions

*a, b, c, d may be numbers or variable expressions.*

<table>
<thead>
<tr>
<th>Operation</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adding and Subtracting</td>
<td>( \frac{a}{b} + \frac{c}{d} = \frac{ad + cd}{bd} )</td>
</tr>
<tr>
<td>Requires</td>
<td>a Common Denominator</td>
</tr>
<tr>
<td>Multiplying</td>
<td>( \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} )</td>
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<tr>
<td>just multiply tops/bottoms</td>
<td></td>
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<tr>
<td>Dividing - ”flip” bottom</td>
<td>( \frac{a/b}{c/d} = \frac{a \cdot d}{b \cdot c} )</td>
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<tr>
<td>Reducing Fractions</td>
<td>( \frac{a \cdot c}{b \cdot c} = \frac{a}{b} )</td>
</tr>
<tr>
<td>cancel common terms</td>
<td>( \frac{a \cdot c + b \cdot c}{d \cdot c} = \frac{a + b}{d} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{a \cdot c}{b \cdot c + d \cdot c} = \frac{a}{b + d} )</td>
</tr>
</tbody>
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Handout: Fractions and Rational Expressions

Simplify and Reduce the following Fractions/Rational Expressions

a) \( \frac{48}{8} \)                     b) \( \frac{21}{24} \)

c) \( \frac{88}{80} \)                   d) \( \frac{76}{40} \)

e) \( \frac{19 + 7}{7 + 6} \)          f) \( \frac{284 + 4}{271 + 1} \)

\[ g) \frac{21x - 21}{35x + 14} \quad h) \frac{6x + 27}{9x - 21} \]

\[ i) \frac{48x^3 - 84x^2}{42x^3 - 90x^2} \quad j) \frac{54x^2 - 114x}{78x^2 + 6x} \]

\[ k) \frac{56x - 49}{96x - 84} \quad l) \frac{14x + 10}{105x + 75} \]

Convert the Improper Fractions to Mixed Numbers

a) \( \frac{42}{19} \)                     b) \( \frac{23}{19} \)                     c) \( \frac{44}{21} \)

d) \( \frac{43}{13} \)                   e) \( \frac{53}{9} \)                   f) \( \frac{35}{17} \)

Answers a) 6; b) \( \frac{7}{9} \); c) \( \frac{11}{19} \); d) \( \frac{19}{17} \); e) 2; f) \( \frac{18}{17} \); g) \( \frac{3(x-1)}{6x+2} \); h) \( \frac{2x+9}{32-7} \); i) \( \frac{14-8x}{15-7z} \); j) \( \frac{8x-10}{13x+1} \); k) \( \frac{7}{12} \); l) \( \frac{2}{15} \); a) \( 2 \frac{4}{10} \); b) \( 1 \frac{4}{19} \); c) \( 2 \frac{2}{21} \); d) \( 3 \frac{4}{15} \); e) \( 5 \frac{5}{9} \); f) \( 2 \frac{1}{17} \)
Adding and Subtracting

a) \(\frac{5}{23} + \frac{6}{17}\)

b) \(\frac{6}{7} + \frac{8}{23}\)

c) \(\frac{12}{23} - \frac{15}{13}\)

d) \(\frac{8}{3} - \frac{13}{19}\)

e) \(\frac{15}{4} + \frac{17}{4} - \frac{290}{40}\)

f) \(\frac{10}{7} + 1 - \frac{60}{35}\)

g) \(\frac{4}{5} + \frac{3}{1 + x}\)

h) \(\frac{5}{7} + \frac{4}{2 + x}\)

i) \(\frac{2}{x + 5} + \frac{1}{-3 + x}\)

j) \(\frac{1}{x + 4} + \frac{5}{6 + x}\)

Multiplying

a) \(\left(\frac{1}{11} + \frac{1}{7}\right) \cdot \left(\frac{1}{7} - \frac{1}{11}\right)\)

b) \(\left(\frac{3}{5} + \frac{3}{11}\right) \cdot \left(\frac{6}{11} - \frac{3}{5}\right)\)

c) \(\left(x + \frac{3}{2}\right) \cdot \left(\frac{5}{2} - x\right)\)

d) \(\left(x + \frac{7}{3}\right) \cdot \left(\frac{7}{3} - x\right)\)

Answers a) \(\frac{223}{391}\); b) \(\frac{104}{161}\); c) \(-\frac{189}{299}\); d) \(\frac{113}{57}\); e) \(\frac{3}{4}\); f) \(\frac{5}{7}\); g) \(\frac{4x+19}{5(x+1)}\); h) \(\frac{5x+38}{7(x+2)}\);
i) \(\frac{3x-1}{(x-3)(x+5)} = \frac{3x-1}{x^2+2x-15}\); j) \(\frac{2(3x+13)}{(x+4)(x+6)} = \frac{6x+26}{x^2+10x+24}\);

Answers a) \(\frac{72}{3025}\); b) \(-\frac{144}{3025}\); c) \(-x^2 + x + \frac{15}{4}\); d) \(\frac{49}{3} - x^2\);
More Complicated Fractions

\( a) \frac{3}{7} + \frac{5}{1 + \frac{3}{4}} \quad b) \frac{2}{5} + \frac{3}{5 + \frac{3}{4}} \)

\( c) \frac{4}{7} + \frac{1}{2 + \frac{2}{x+4}} \quad d) \frac{5}{4} + \frac{3}{5 + \frac{2}{x+7}} \)

\( e) \frac{1 + \frac{3}{2}}{\frac{10}{3} + 6} \quad f) \frac{8 + \frac{5}{12}}{\frac{4}{5} + 7} \)

\( g) \frac{6 + \frac{3}{x}}{\frac{4}{x} + 7} \quad h) \frac{5 + \frac{5}{x}}{\frac{4}{x} + 3} \)

Don’t forget to reduce the fractions.

\( a) \frac{3}{5} - \frac{5}{11} + \frac{3}{\frac{55}{5}} \quad b) \frac{5}{7} - \frac{8}{11} + \frac{100}{77} \)

\( c) \frac{4}{z + 3} + \frac{5}{z - 11} + \frac{z^2 - 18z + 7}{(z - 11)(z + 3)} \quad d) \frac{1}{z + 11} + \frac{8}{z - 7} + \frac{z^2 - 12z - 109}{(z - 7)(z + 11)} \)

Answers a) \( \frac{23}{7} \); b) \( \frac{22}{25} \); c) \( \frac{15x+68}{14(x+5)} \); d) \( \frac{37x+269}{4(5x+37)} \); e) \( \frac{15}{56} \); f) \( \frac{505}{468} \); g) \( \frac{3(2x+1)}{7x+4} \); h) \( \frac{5(x+1)}{3x+4} \);

Answers a) \( \frac{1}{5} \); b) \( \frac{9}{7} \); c) \( \frac{x+2}{x+3} \); d) \( \frac{x+4}{x+11} \);