What are $\dfrac{0}{0}$ and $\dfrac{\infty}{\infty}$?

Answer: M.W. = MORE WORK The problem is NOT DONE .
The more work is often **Algebra**!

These expressions mean More Work.

$\mathbf{\dfrac{0}{0}}$	$\boldsymbol{\dfrac{\infty}{\infty}}$	$\mathbf{0 \cdot \infty}$
$\boldsymbol{\infty - \infty}$	$\mathbf{0 \cdot \infty}$	$\mathbf{0^{\infty}}$

You should know the following *(# denotes a non-zero number)*

When you Plug-in and get these - you are done.

a) $\dfrac{0}{\#} = 0$	b) $\dfrac{\#}{0} = \infty$	c) $\dfrac{0}{\infty} = 0$
d) $\dfrac{\#}{\infty} = 0$	e) $\dfrac{\infty}{\#} = \infty$	f) $\dfrac{\infty}{0} = \infty$

You should think of ∞ As A Number. It is a legitimate answer.

Recall: What is the value of the expression $\dfrac{2+x}{x-1}$ when $x=5$?

Answer: $\dfrac{2+x}{x-1} \Longrightarrow \dfrac{2+5}{5-1} = \dfrac{7}{4}$ *Just plug in. Get a number* $\Rightarrow$ *done.*

Example: What is the value of the expression $\dfrac{x^2-3x+2}{x^2-2x+1}$ when $x=1$?

Attempt: $\Longrightarrow \dfrac{1-3+2}{1-2+1} = \dfrac{0}{0}$ *– Need to do MORE WORK*

Example: What is the value of the expression $\dfrac{2x^2+3x}{4x+5x^2}$ when $x=\infty$?

Attempt: $\dfrac{2x^2+3x}{4x+5x^2} \Longrightarrow \dfrac{2\cdot\infty^2+3\cdot\infty}{4\cdot\infty+5\cdot\infty} = \dfrac{\infty}{\infty}$ *– Need to do MORE WORK*

Example: What is the value of the expression $\dfrac{2x^2+3x}{4x+5x^2}$ when $x=0$?

Attempt: $\dfrac{2x^2+3x}{4x+5x^2} \Longrightarrow \dfrac{2\cdot 0^2+3\cdot 0}{4\cdot 0+5\cdot 0} = \dfrac{0}{0}$ *– Need to do MORE WORK*

Example: What is the value of the expression $\dfrac{x^2-3x+2}{x^2-2x+1}$ when $x=1$?

Technique: Factor - Cancel - Plugin - Get Answer

$$\begin{aligned}
\frac{x^2-3x+2}{x^2-2x+1} &= \frac{(x-1)(x-2)}{(x-1)(x-1)} && \text{Factor} \\
&= \frac{(x-2)}{(x-1)} && \text{Cancel} \\
&= \frac{(1-2)}{(x-1)} = \frac{-1}{0} && \text{Plug in} \\
&= -\infty && \text{Answer}
\end{aligned}$$

THIS is a legitimate answer!

Example: Evaluate $\dfrac{2x^2+3x}{4x+5x^2}$ at $x=\infty$?

Technique: Divide by a Power of x - Plug In - Get Answer

$$\frac{2x^2+3x}{4x+5x^2}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{2+\frac{3}{x}}{\frac{4}{x}+5}$$

Divide by **"highest exponent"** of x in denominator

$$= \frac{2+\frac{3}{\infty}}{\frac{4}{\infty}+5} = \frac{2}{5}$$

Use $\boldsymbol{\frac{\#}{\infty}=0}$

Example: Evaluate $\dfrac{2x^2+3x}{4x+5x^2}$ at $x=0$?

Technique: Divide by a Power of x - Plug In - Get Answer

$$\frac{2x^2+3x}{4x+5x^2}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \frac{2x+3}{4+5x}$$

Divide by **"lowest exponent"** of x in denominator

$$= \frac{2\cdot 0+3}{4+5\cdot 0} = \frac{3}{4}$$

Difference quotient example 1

Example: Evaluate $\dfrac{f(x+h)-f(x)}{h}$ at $h=0$ for $f(x)=2x+3$

Note: If you plug in $h=0$ first, you will get $\dfrac{f(x+0)-f(x)}{0}=\dfrac{0}{0}$

Technique: Use Algebra to Simplify - Cancel an h - Get Answer

$$\frac{f(x+h)-f(x)}{h}=\frac{\big(2(x+h)+3\big)-\big(2x+3\big)}{h}$$

$$=\frac{2x+2h+3-2x-3}{h}$$

$$=\frac{2h}{h}$$

$$=2$$

Digression - Recall

$f(x)$ is Pronounced "f of x".

It does Not mean "f times x"

Digression - Recall

$f(x) = x^2 + 3x + 1$

SAYS
USE THE EXPRESSION $x^2 + 3x + 1$
TO CALCULATE THE VALUE of $f(x)$

Illustration

$$
\begin{array}{rcll}
\text{f(3)} = & 3^2 + 5 \cdot (3) + 1 & = 9 + 15 + 1 & = 25 \\
\text{f(-2)} = & (-2)^2 + 5 \cdot (-2) + 1 & = 4 - 10 + 1 & = -5 \\
\text{f(x+h)} = & (x+h)^2 + 5 \cdot (x+h) + 1 & & \\
= & x^2 + 2xh + h^2 + 5x + 5h + 1 & &
\end{array}
$$

To Calculate $f(3)$ - Substitute 3 into the Expression - get 25.

To Calculate $f(-2)$ - Substitute -2 into the Expression - get -5.

To Calculate $f(x+h)$ - Substitute $x+h$ into the Expression - get the above answer.

Digression

The Difference Quotient is: $\dfrac{f(x+h)-f(x)}{h}$.

It means - You will be GIVEN an Expression for $f(x)$.

Then, CALCULATE $f(x+h)$ - and get a another expression.

Then, SUBTRACT the two expressions

Then, DIVIDE by h

Difference quotient example 2

Example: Evaluate $\dfrac{f(x+h)-f(x)}{h}$ at $h=0$ for $f(x)=x^2+5x+3$

Technique: Use Algebra to Simplify - Cancel an h - Get Answer

$$\frac{f(x+h)-f(x)}{h} = \frac{\big((x+h)^2+5(x+h)+3\big)-\big(x^2+5x+3\big)}{h}$$

$$= \frac{x^2+2xh+h^2+5x+5h+3-x^2-5x-3}{h}$$

$$= \frac{2xh+h^2+5h}{h}$$

$$= 2x+h+5 \quad \textit{Canceling}$$

$$= 2x+5$$

NOTICE - the answer may still have "x" in it. (It does not have to be a number as in the previous example.)