Exponent and Radicals - Rules for Manipulation

Algebraic Rules for Manipulating Exponential and Radicals Expressions.

In the following, n, m, k, j are arbitrary -
. *they can be integers or rationals or real numbers.*

$$\frac{b^n \cdot b^m}{b^k} = b^{n+m-k}$$	Add exponents in the numerator and Subtract exponent in denominator.
$$\left(\frac{a^n \cdot b^m}{c^k}\right)^j = \frac{a^{n\cdot j} \cdot b^{m\cdot j}}{c^{k\cdot j}}$$	The exponent outside the parentheses Multiplies the exponents inside.
$$\left(\frac{a^n}{b^m}\right)^{-1} = \frac{b^m}{a^n}$$	Negative exponent "flips" a fraction.
$b^0 = 1$ $\quad$ $b = b^1$	Don't forget these

Convert Radicals to Exponent notation

$$\sqrt{a} = a^{1/2}$$
$$\sqrt[m]{a} = a^{1/m}$$
$$\sqrt[m]{a^n} = a^{n/m}$$

Radicals - Reducing

$\sqrt{a^2 \cdot b} = a\sqrt{b}$ $\sqrt[m]{a^m \cdot b} = a\sqrt[m]{b}$	Remove squares from inside

Exponent and Radicals - Solving Equations

$x^{n/m} = y \Leftrightarrow x = y^{m/n}$	Solve a power by a root Solve a root by a power

Example

a) Simplify $\left(\frac{2}{5}\right)^3$

Method $$\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{2\cdot 2\cdot 2}{5\cdot 5\cdot 5} = \mathbf{\frac{8}{125}}$$

b) Simplify $\left(\frac{2\cdot 3^2}{5^3}\right)^2$

Method $$\left(\frac{2\cdot 3^2}{5^3}\right)^2 = \frac{2^2\cdot 3^{2\cdot 2}}{5^{3\cdot 2}} = \frac{4\cdot 81}{15,625} = \mathbf{\frac{324}{15,625}}$$

Illustration: where is the negative?

c) Simplify $\big(-3\big)^4$ (the 'negative' is inside the parentheses)

Method $$\big(-3\big)^4 = (-3)\cdot(-3)\cdot(-3)\cdot(-3) = \mathbf{81}$$

d) Simplify $-\big(3\big)^4$ (the 'negative' is outside the parentheses)

Method $$-\big(3\big)^4 = -(3)\cdot(3)\cdot(3)\cdot(3) = -\mathbf{81}$$

e) Simplify $\left(\frac{2}{5}\right)^{-3}$ (the 'negative' is in the exponent)

Method $$\left(\frac{2}{5}\right)^{-3} = \frac{1}{(2/5)^3} = (or = \left(\frac{5}{3}\right)^3) = \frac{5^3}{2^3}$$

$$= \mathbf{\frac{125}{8}}$$

More Examples

f) Simplify $\dfrac{x^3 \cdot x^7}{x^5}$

Method $$\frac{x^3 \cdot x^7}{x^5} = x^{3+7-5} = \mathbf{x^5}$$

g) Simplify $(2a^3b^2)(3ab^4)^3$

Method $$(2a^3b^2)(3ab^4)^3 = 2a^3b^2 \cdot 3^3a^3b^{4\cdot 3}$$

$$= (2 \cdot 27)(a^{3+3})(b^{2+12})$$

$$= \mathbf{54a^6b^{14}}$$

h) Simplify $\left(\dfrac{x}{y}\right)^3\left(\dfrac{y^2x}{z}\right)^4$ (*give answer with only positive exponents*)

Method $$\left(\frac{x}{y}\right)^3\left(\frac{y^2x}{z}\right)^4 = \frac{x^3}{y^3} \cdot \frac{y^{2\cdot 4}x^4}{z^4}$$

$$= \frac{x^{3+4}y^{8-3}}{z^4} = \mathbf{\frac{x^7y^5}{z^4}}$$

More Examples with negatives

i) Simplify $\dfrac{6st^{-4}}{2s^{-2}t^{2}}$ (*give answer with only positive exponents*)

Negative exponents flip location: A negative exponent in the numerator moves to the denominator. And a negative exponent in the denominator moves to the numerator.

Method $$\frac{6st^{-4}}{2s^{-2}t^{2}} = \frac{6ss^{2}}{2t^{4}t^{2}} = \mathbf{\frac{3s^{3}}{t^{6}}}$$

j) Simplify $\left(\dfrac{y}{3z^{3}}\right)^{-2}$ (*give answer with only positive exponents*)

A Negative exponent 'flips' the fraction.

Method $$\left(\frac{y}{3z^{3}}\right)^{-2} = \left(\frac{3z^{3}}{y}\right)^{2} = \mathbf{\frac{9z^{6}}{y^{2}}}$$

More Examples

k) Simplify $\dfrac{(2x^3)^2(3x^4)}{(x^3)^4}$

Method $\quad \dfrac{(2x^3)^2(3x^4)}{(x^3)^4} = \dfrac{2^2x^{3\cdot 2}\cdot 3x^4}{x^{3\cdot 4}}$

$$= \frac{(4\cdot 3)x^6x^4}{x^{12}} = 12x^{6+4-12} = \mathbf{\frac{12}{x^2}}$$

Examples Simplifying Roots

a) Simplify $\sqrt{8}$

Method $$\sqrt{8} = \sqrt{4 \cdot 2} = \mathbf{2\sqrt{2}}$$

b) Simplify $\sqrt{75}$

Method $$\sqrt{75} = \sqrt{25 \cdot 3} = \mathbf{5\sqrt{3}}$$

c) Simplify $\sqrt[3]{x^4}$

Method $$\sqrt[3]{x^4} = \sqrt[3]{x^3 \cdot x} = \mathbf{x}\sqrt[3]{x}$$

d) Simplify $\sqrt[4]{81x^8y^4}$

Method $$\sqrt[4]{81x^8y^4} = \sqrt[4]{81}\sqrt[4]{x^8}\sqrt[4]{y^4} = \mathbf{3x^2y}$$

Digression: Technically $\sqrt{x^2} = |x|$ *and* $\sqrt[4]{x^4} = |x|$ *but we will not worry about that at this time.*

More Examples

e) Simplify $\sqrt{32} + \sqrt{200}$

Method
$$\begin{aligned}\sqrt{32} + \sqrt{200} &= \sqrt{16 \cdot 2} + \sqrt{100 \cdot 2} \\ &= 4\sqrt{2} + 10\sqrt{2} = \mathbf{14\sqrt{2}}\end{aligned}$$

f) Simplify $\sqrt{25b} - \sqrt{b^3}$

Method
$$\begin{aligned}\sqrt{25b} - \sqrt{b^3} &= \sqrt{25 \cdot b} + \sqrt{b^2 \cdot b} \\ &= 5\sqrt{b} - b\sqrt{b} = (\mathbf{5} - \mathbf{b})\sqrt{\mathbf{b}}\end{aligned}$$